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RESEARCH ARTICLE

An Investigation of 5-Year-Old Children's Reasoning About Probability

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Abstract

This intrinsic case study was conducted to investigate 5-years-old children's thoughts about probability. In this context, children's predictions about probabilistic trials, their choices on probabilistic trials, inferences about the consequences of probabilistic trials, reasonings about complementary events, considerations on the recurrence of trials, and mathematical appropriateness of children's responses were detailly investigated. Twenty-four children (63- to 70-month-olds) participated in this study. A progressive game played with the help of wheel mechanisms, a short story about probabilistic situations, and a set having interview-based procedures (Probabilistic Trials for Five-Year-Old Children) were used as data collection tools. Summative content analysis was used to analyze the data. Children's thoughts about probability were categorized under different categories. Coefficient of interrater reliability was found to be as .87 according to Miles and Huberman's formula. The results of this study show us that 5-year-old children have mathematically appropriate understandings about uncertain events, probable events, other possible factors affecting probability, and quantifying probability. Besides, the children who participated in the study were having some misconceptions on probability, such as focusing on the consequences of events, "Emotional decision-making," "Assuming previously observed outcome will repeat," and "Assuming increased probability as certain event." As another result, most of their inferences about probabilistic situations were not consistent.

Keywords: Children, early mathematics education, probability, probabilistic thinking, reasoning

Introduction

Probability is a special tool for people to make reasonable decisions, to predict and infer about various probabilistic situations (Jones et al., 2007). Probability is a branch of mathematics. It makes no claims to the real world and its conditions, although it originated from gambling problems. It has its own phenomenon and mathematical explanations. However, its phenomenon and explanations are applicable in the real world and in human life. Many real-world problems are applicable and explicable with the help of probability (Athreya, 2015).

Probability is in everyone's life, even though they do not know much about it. Somehow everyone encounters with probability, makes decisions and reasonings about it. So how do they do this? At this point, the universal competencies of being human come into play. Everyone may experience the wetness of the rain, coldness of low temperature, or the force of gravity. Thus, anyone can predict the fate of an object in the rain or at low temperatures, or of an object thrown into the air, based on their past experiences. Everyone will probably infer it was wet, frozen, or dropped (Johnson-Laird, 1994). According to Piaget and Inhelder, children's probabilistic thinking depends on combinatory operational systems. They assert that children should have an "anticipatory schema" to develop understanding of probability and chance. This schema means children's going beyond the fact of previous events. And it allows children to predict unexpected events or situations in the future (Piaget & Inhelder, 1951, as cited in, Carlson, 1970). Piaget and Inhelder define three developmental stages for probabilistic cognition. According to this definition, children up to 8 years (stage 1) do not have adequate knowledge about the concept of probability. And they also cannot distinguish between casual and random events. Children between the ages of 8 and 12 (Stage 2) can develop a basic understanding of quantifying probability. They realize the difference between certain and uncertain events. Children from 12 years (Stage 3) can make more accurate probabilistic calculations, and they can make deductive reasoning on probability (Piaget & Inhelder, 1975, as cited in, Gong & He, 2017). On the contrary, Bryant and Nunes (2012) state that even very young children have some knowledge and understanding about probability. Preschool age children can develop understandings about randomness, consequences of events, the sample space, quantifying and comparing the probabilities, and correlations. Their prior knowledge and experiences about probability are the foundation of probabilistic thinking.

There are relevant studies on different-age group-children's understanding of probability and their probabilistic thinking skills in the current literature. Sari et al. (2017) studied with children attending elementary school by using wheels having two differently colored equal circle slices. This study revealed that children's probabilistic thinking depends on their math abilities. And it also revealed that children are pretty good at sample spaces, probability of an event, and comparison of probabilities. In HodnikČadež and Škrbec's (2011) study, 4- to 8-year-old children's understandings of the concepts related to probability were examined. Their study revealed that there was no statistically significant difference on children's understanding of probability across different age levels, although it is more consistent at age 8. According to this study, children have some difficulties on predicting the equal probabilities and probabilistic correlations of independent events. However, they stated that children may understand and recognize the certain events, possible events, and improbable events. Bayless and Schlottmann

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(2010) played a task-based game with 5–7-year-old children to find out children's thoughts on expected value and probability. They expected children to roll three marbles toward large-, medium-, and small-sized gates from three different distances. Their study reveals that children consider the possibilities in order to successfully accomplish the task. Children's judgements depend on the difficulty level. In their study, children tended to choose the large and proximate gate to increase the possibility of winning the game. Vásquez-Ortiz and Alsina (2019) studied with 4–6-year-old children. Their study revealed that children use simple mathematical language about probability. It was also revealed that children have some prior knowledge about sample space, possibility of an event, and comparison of possibilities. According to Girotto and Gonzalez (2008), 5-year-old children may revise their decisions and judgments into probabilistically appropriate ones in the repetition of probabilistic situations. Pange (2002) states that 5-year-old children develop appropriate understandings on the possible outcomes of probabilistic situations. Tatsis et al. (2008) discussed the fairness of probabilistic games with 5-year-old children. This study revealed that children are capable to overcome their intuitions about the fairness of the game. It also revealed that children's thoughts about probability are mostly based on the counting strategy. Sánchez-Robayo and Wilkins's (2020) case study, which was conducted with 4-year-old children, revealed that children have preoperational understanding against the changes of sample space in nonreplacement probabilistic situations. It also revealed that children partially understand independent events and their judgments related to probability are subjective, inconsistent, and based on quantity. Denison and Xu's (2010) study revealed that 12–14-month-old infants can predict single-event probability significantly better than chance. Davis et al. (2011) studied infants by using the eye movement paradigm. Their study revealed that even 10-month-old infants show evidence of probability. Xu and Garcia's (2008) study revealed that 8-month-old infants can infer about which population the outcoming items belong to, in probabilistic trials.

Children may have some understanding and prior knowledge about probability, but they are dependent on their experiences, language, belief, culture, and individual differences (Amir & Scott-Williams, 1999). Moreover, children may have misconceptions on probability because of their having limited experiences and perceptions. Jun and Pereira-Mendoza (2002) state that children have many misconceptions on probability such as "probability cannot be calculated," "interpreting the probability based on outcomes," "considering probability as certainty," "interpreting the probabilistic situations based on subjective judgments," "ignoring the probabilistic correlations," "the repetition of probabilistic situations will complicate the interpretation," "the complementary event will occur in the case of repetition of probabilistic situation," "developing a subjective probability calculation method." Primi et al. (2019) state that probabilistic thinking is related to many different learning areas and academic achievement. They emphasize that children's having misconceptions on probability has negative effects on development of probabilistic thinking. They also state that educational implementations will contribute the development of children's probabilistic thinking. Therefore, they see determining children's thoughts, understanding and misconceptions on probability as important facts.

Probabilistic thinking has become an important thinking skill of this century. Nowadays, individuals should make choices between various types of information and opportunities. They also should consider the uncertainty, make predictions, and develop alternative ways or solutions against various situations. For children, these situations occur in their daily life, especially in their play. Children use probabilistic thinking while they are choosing toys to play, determining the chance of winning a game, playing any conditional progressive game by using dice or wheels (Tatsis, Kafoussi & Skoumpourdi, 2008). According to the National Council of Teachers of Mathematics (2000) standards, probability is a subject area to be taught from pre-kindergarten level. Children are expected to be capable of understanding and using the basic concepts of probability.

Considering the importance of determining children's thoughts and misconceptions on probability for their learning and achievement (Primi et al., 2019) and for planning educational applications (Vásquez Ortiz & Alsina, 2019), it is essential to determine children's thoughts about probability. In this study, 5-year-old children were expected to reflect their thoughts by making predictions and choices, inferencing, reasoning, and considering the probabilistic trials.

Methods

This study was conducted as an intrinsic case study. An intrinsic case study allows researchers to gather deeper information about specific groups (Hancock & Algozzine, 2017). In this study, 5-year-old children's thoughts about probabilistic trials were detailly investigated.

Participants

Children of 63–70 months of age (mean age: 65.7 months) participated in this study. A total of 24 children (13 girls and 11 boys) were selected based on the convenience sampling method. This sampling method ensures researchers to practically reach the new or current groups. Also, it allows them to practically implement the research procedures (Creswell, 2012). In this study, choosing preschool educational institutions being so close and accessible to the researcher's institution was decisive.

Data Collection Tools

A set titled "Probabilistic Trials for Five-Year-Old Children" was used as a data collection tool. This set was developed by the researcher of this study. This tool is for determining 5-year-old children's thoughts, predictions, inferences, and considerations about probability. The components of the set are listed as follows:

- Two different wooden wheels for experiencing the probabilistic situations
- A short story titled "Siyo & Beyo the Puppies" for explaining the rules
- A progressive game titled "Let's Get the Puppies to the Bone"
- Five different semistructured interview questions
- A written record form

Four equal circle slices with wooden wheel mechanisms are placed on wooden platforms. Children would spin them by hand. An arrow would indicate which color has come out. Mechanism 1 has three white and one black-colored circle slices, while mechanism 2 has two white and two black-colored circle slices. The mechanisms are shown in Figure 1.

The short story is as follows: *"There were two puppies. Black and white puppies. Do you know, puppies like bones so much? These two puppies, too. Let's play a game and help them to reach the bone. You need to choose one of the puppies. You may explain your choice by selecting white- or black-colored paw. We will spin the wheel. The puppy of the color that comes out will advance one step. We will advance our puppies by using the paws. Let's turn and see which color will come out. The puppy that leads gets the bone (wins the game). Let's see which colored puppy will reach the bone. Are you ready?"*

A symbolic presentation of the instruments of game is shown on Figure 2. This game is being played as reminded by short story. Each puppy follows its own way to reach the bone. Mechanisms 1 and 2 are separately used to advance in this game. (Game is played for twice)

Figure 1.
Wooden Wheel Mechanisms 1 and 2.

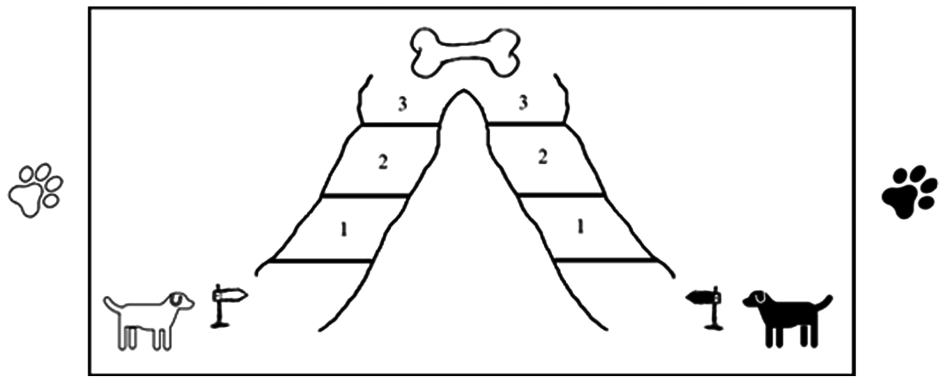

Figure 2.
Paws and Tray of the Game Titled "Let's Get the Puppies to the Bone."

The semistructured interview questions are listed as follows:

- Which one of these puppies do you think would win the game if we use this wheel? Why?
- What colored puppy would you choose to win the game? Why?
- What colored puppy won the game? Why do you think such a result came out?
- Do you think the other puppy would have won the game, too? Why?
- What color puppy do you think will win the game if we play it again? Why?

A written record form in which children's responses and progression of the game were recorded was used. An example of this form is presented in Appendix 1.

Data Collection Procedures

All the necessary permissions (legal and ethical) were obtained before conducting this study. Parents of children were enlightened about the content of the study and their approval were obtained. Children were also enlightened about the same and were expected to participate voluntarily. Children's comfort and feelings of being secure were considered. To ensure that, they were offered several choices about where to hold the interviews. Choices were having the interview in waiting room, in classroom or in entrees. Interactions between other children during the interviews were prevented. Data collection tools were introduced to each child and children were free to try and experience them for a while. Children were told a story titled "Siyo & Beyo the Puppies" to remind the rules of implementations. Then the game titled "Let's Get the Puppies to the Bone" was played by using mechanisms 1 and 2. Children's responses were recorded on to the written record form, before, during, and at the end of the game. Children were free to express as many opinions as they wish, to change their response, and not to respond. A total of 285 responses were recorded during procedures.

Data Analysis

Seven responses of children were about not having any idea; that is why only 278 responses out of 285 were analyzed by two different coders. Participants were given pseudonyms as participant numbers from P1 to P24. Summative content analysis was used to analyze the data. This technique allows researchers to examine written data more effectively in a simple manner. And it also allows to categorize the data clearly (Hsieh & Shannon, 2005; Rapport, 2010). Written record forms were examined, and children's responses were categorized under different categories. All the categories were examined in the context of mathematical appropriateness. Categories and sample expressions of children are presented in Appendix 2.

Miles and Huberman's (1994) formula, mathematically represented as $R=[\text{Consensus}/(\text{Consensus}+\text{Dissidence})]$, was used to calculate the coefficient of interrater reliability. According to this formula, the coefficient is expected to be at least .70. In this study, two different coders analyzed the data and the coefficient was calculated as $R=[244/(244+34)=.87$.

Persuasiveness

Some measures were taken during all the steps of this study. Only one researcher implemented the data collecting procedures to ensure the consistency of the data. Children were let free to decide whether to participate. They were free to try, examine, and use the tools to eliminate their hesitations before the implementations. Children were offered several choices to make them feel secure. Interactions between peers were prevented to eliminate any influences. Children were allowed to spin the wheel and to advance the paws on their own. Wheels were spun by hand to ensure the randomness, instead of the programmed mechanisms giving firm results. Two different mechanisms with black and white circle slices of different proportions were used to ensure the chance of all probable sequences come out. Children were free to express as many opinions as they wish, to change their minds, to express their reasons, to return to previous questions, or not to express any idea. Two different coders analyzed the data to obtain interrater reliability.

Findings

Findings on children's predictions about probabilistic trials, choices on probabilistic trials, inferences about the consequences of probabilistic trials, reasoning about complementary events, considerations on the repetition of probabilistic trials, and mathematical appropriateness of children's thoughts in general manner are presented in this section.

Children's Predictions About Probabilistic Trials

Children were expected to predict which colored puppy would win the game. Children's thoughts for the events of mechanisms 1 and 2 were categorized as presented in Table 1. According to this table, most of children's responses for the events of mechanism 1 (45.83%) refer to "Quantifying the probability." 16.66% of them refer to "Emotional decision-making," while 12.50% of them refer to Expressing no idea. 8.33% of them refer to "Assuming previously observed outcome will repeat" and "Emphasizing uncertain event." Lastly 4.16% of them refer to "Assuming the higher probability as certain event" and "Emphasizing probable event."

Considering the Table 1, most of children's responses for the events of mechanism 2 (58.62%) refer to "Quantifying the probability." 13.79% of them refer to "Assuming increased probability as certain event." After then, 6.89% of them refer to "Assuming previously observed outcome will repeat and assuming complementary events will occur in sequence." Lastly, 3.44% of them refer to "Considering other possible factors affecting probability," "Emphasizing probable event," "Emphasizing uncertain event," and "Expressing no idea."

Table 1.
Children's Predictions About Probabilistic Trials

Mechanism	Categories	*f*	%
Mechanism 1	Quantifying the probability	11	45.83
	Emotional decision-making	4	16.66
	Expressing no idea	3	12.50
	Assuming previously observed outcome will repeat	2	8.33
	Emphasizing uncertain event	2	8.33
	Assuming the higher probability as certain event	1	4.16
	Emphasizing probable event	1	4.16
	Total	24	100
Mechanism 2	Quantifying the probability	17	58.62
	Assuming increased probability as certain event	4	13.79
	Assuming previously observed outcome will repeat	2	6.89
	Assuming complementary events will occur in sequence	2	6.89
	Considering other possible factors affecting probability	1	3.44
	Emphasizing probable event	1	3.44
	Emphasizing uncertain event	1	3.44
	Expressing no idea	1	3.44
	Total	29	100

Table 2.
Children's Choices on Probabilistic Trials

Mechanism	Categories	*f*	%
Mechanism 1	Quantifying the probability	12	48.00
	Emotional decision-making	8	32.00
	Assuming the higher probability as certain event	2	8.00
	Assuming previously observed outcome will repeat	2	8.00
	Expressing no idea	1	4.00
	Total	25	100
Mechanism 2	Emotional decision-making	8	30.77
	Quantifying the probability	4	15.38
	Assuming previously observed outcome will repeat	4	15.38
	Assuming increased probability as certain event	3	11.54
	Emphasizing probable event	3	11.54
	Assuming complementary events will occur in sequence	2	7.69
	Considering other possible factors affecting probability	1	3.85
	Assuming previously observed outcome as certain event	1	3.85
	Total	26	100

Children's Choices on Probabilistic Trials

Children were expected to choose one of the puppies to win the game and also to explain the reason of their choice. Children's thoughts for the events of mechanisms 1 and 2 were categorized as shown in Table 2. As we may see in Table 2, most of the children's responses for the events of mechanism 1 (48.00%) refer to "Quantifying the probability." 32.00% of them refer to "Emotional decision-making." 8.00% of them refer to "Assuming the higher probability as certain event" and "Assuming previously observed outcome will repeat." Lastly, 4.00% of them refer to "Expressing no idea."

Considering the Table 2, most of children's responses for the events of mechanism 2 (30.77%) refer to "Emotional decision-making." After that "Quantifying the probability" and "Assuming previously observed outcome will repeat" come out in equal proportion (15.38%). Of them, 11.54% refer to "Assuming increased probability as certain event" and "Emphasizing probable event." As high as 7.69% of them refer to "Assuming complementary events will occur in sequence." Lastly, 3.85% of them refer to "Considering other possible factors affecting probability" and "Assuming previously observed outcome as certain event."

Children's Inferences About the Consequences of Probabilistic Trials

The children were asked the question, "Why do you think such a result came out?" Children's inferences about the consequences of probabilistic trials were examined. Categories reached as result of the examination for mechanisms 1 and 2 were presented in Table 3. This table shows us that most of children's responses for the events of mechanism 1 (51.72%) refer to "Quantifying the probability." "Decision-making based on the consequences of events" comes out in a proportion of 31.03%. Afterward 10.34% of them refer to "Considering other possible factors affecting probability." Lastly, 3.45% of them refer to "Assuming the higher probability as certain event and emotional decision-making."

According to Table 3, most of children's responses for the events of mechanism 2 (35.71%) refer to "Decision-making based on the consequences of events." This result is followed by "Considering other possible factors affecting probability," "Quantifying the probability and

Table 3.
Children's Inferences About the Consequences of Probabilistic Trials

Mechanism	Categories	f	%
Mechanism 1	Quantifying the probability	15	51.72
	Decision-making based on the consequences of events	9	31,03
	Considering other possible factors affecting probability	3	10.34
	Assuming the higher probability as certain event	1	3.45
	Emotional decision-making	1	3.45
	Total	29	100
Mechanism 2	Decision-making based on the consequences of events	10	35.71
	Considering other possible factors affecting probability	4	14.28
	Quantifying the probability	4	14.28
	Emphasizing probable event	4	14.28
	Assuming increased probability as certain event	3	10.71
	Emphasizing uncertain event	1	3.57
	Emotional decision-making	1	3.57
	Expressing no idea	1	3.57
	Total	28	100

emphasizing probable event," in a proportion of 14.28%. "Assuming increased probability as certain event" comes out in a proportion of 10.71%. Lastly, 3.57% of them refer to "Emphasizing uncertain event," "Emotional decision-making," and "Expressing no idea."

Children's Reasonings About Complementary Events

Children were expected to reflect their ideas about whether the other puppy could have won. Children's thoughts for the events of mechanisms 1 and 2 were examined, and several categories were reached as shown in Table 4. Considering this, most of children's responses for the events of mechanism 1 (30.43%) refer to "Decision-making based on the consequences of events." "Emphasizing probable event" comes out second in a proportion of 26.09%. Following that, 17.39% of them refer to "Quantifying the probability," 15.22% of them "Assuming the lower probability as improbable event," and 6.52% of them "Assuming unobserved outcome as improbable event." Lastly, 2.17% of them refer to "Considering other possible factors affecting probability" and "Expressing no idea."

Table 4.
Children's Reasonings About Complementary Events

Mechanism	Categories	*f*	%
Mechanism 1	Decision-making based on the consequences of events	14	30.43
	Emphasizing probable event	12	26,09
	Quantifying the probability	8	17.39
	Assuming the lower probability as improbable Event	7	15.22
	Assuming unobserved outcome as improbable event	3	6.52
	Considering other possible factors affecting probability	1	2.17
	Expressing no idea	1	2.17
	Total	46	100
Mechanism 2	Emphasizing probable event	12	42.85
	Quantifying the probability	7	25,00
	Assuming previously observed outcome will repeat	2	7.14
	Considering other possible factors affecting probability	2	7.14
	Decision-making based on the consequences of events	2	7.14
	Emphasizing the effects of recurrence of trials	2	7.14
	Emotional decision-making	1	3.57
	Total	28	100

According to Table 4, most of children's responses for the events of mechanism 2 (42.85%) were categorized under the category of "Emphasizing probable event." A quarter of the responds were categorized under the category of "Quantifying the probability." Following that, 7.14% of them were categorized under the categories of "Assuming previously observed outcome will repeat," "Considering other possible factors affecting probability," "Decision-making based on the consequences of events," and "Emphasizing the effects of recurrence of trials." Lastly 3.57% of them were under "Emotional Decision-Making."

Children's Considerations on the Repetition of Probabilistic Trials

The children were asked the question, "Who do you think would win the game if we played it again?" Children's responses were analyzed. Children's considerations on the repetition of trial with mechanisms 1 and 2 were categorized as shown in Table 5. Considering the Table 5, most of children's responses for the events of mechanism 1 (36.00%) refer to "Quantifying the probability." 28.00% of them refer to "Assuming complementary events will occur in sequence," 12.00% of them refer to "Emphasizing probable event." Following that 8.00% of them refer to "Emphasizing uncertain event" and "Assuming previously observed outcome will repeat." Lastly, 4.00% of them refer to "Assuming the higher probability as certain event" and "Considering other possible factors affecting probability."

According to Table 5, most of children's responses for the events of mechanism 2 (24.00%) refer to "Assuming complementary events will occur in sequence." Following that 20.00% of them refer to "Assuming previously observed outcome will repeat," 16.00% of them "Quantifying the probability," and 12.00% of them "Emotional decision-making." Equally, 8.00% of them refer to "Emphasizing probable

Table 5.
Children's Considerations on the Repetition of Probabilistic Trials

Mechanism	Categories	*f*	%
Mechanism 1	Quantifying the probability	9	36,00
	Assuming complementary Events Will Occur in Sequence	7	28,00
	Emphasizing probable event	3	12,00
	Emphasizing uncertain event	2	8,00
	Assuming previously observed outcome will repeat	2	8,00
	Assuming the higher probability as certain event	1	4,00
	Considering other possible factors affecting probability	1	4,00
	Total	25	100
Mechanism 2	Assuming complementary events will occur in sequence	6	24,00
	Assuming previously observed outcome will repeat	5	20,00
	Quantifying the probability	4	16,00
	Emotional decision-making	3	12,00
	Emphasizing probable event	2	8,00
	Emphasizing uncertain event	2	8,00
	Considering other possible factors affecting probability	2	8,00
	Assuming unobserved outcome as improbable event	1	4,00
	Total	25	100

event," "Emphasizing uncertain event," and "Considering other possible factors affecting probability." Lastly, 4.00% of them refer to "Assuming unobserved outcome as improbable event."

Mathematical Appropriateness of Children's Thoughts

In general manner, all the children's responses for mechanisms 1 and 2 were categorized as they are mathematically appropriate or not. Five different categories for mathematically appropriate ones, and nine different categories for inappropriate ones were reached. As we may see in the Table 6, most of the children's responses (55.39%) were mathematically appropriate ones. Besides 44.60% of them were mathematically inappropriate, that is, they carry misconceptions.

Considering the Table 6, most of children's mathematically appropriate responses (59.09%) were categorized under the category of "Quantifying the probability." As high as 24.67% of them were categorized under the category of "Emphasizing probable event, 9.74% of them "Considering other possible factors affecting probability," and 5.19% of them "Emphasizing uncertain event." Lastly, 1.29% of them under "Emphasizing the effects of recurrence of trials." As we may see in Table 6, most of children's mathematically inappropriate responds (28.22%) were categorized under the category of "Decision-making based on the consequences of events." As high as 20.96% of them were categorized under the category of "Emotional decision-making," 15.32% of them "Assuming previously observed outcome will repeat," 13.70% of them "Assuming complementary events will occur in sequence," 8.06% of them "Assuming increased probability as certain event," 5.64% of them "Assuming the lower probability as improbable event," 4.03% of them "Assuming the higher probability as certain event," and 3.22% of them "Assuming unobserved outcome as improbable event." Lastly, 0.80% of them under "Assuming previously observed outcome as certain event."

Table 6.
Mathematical Appropriateness of Children's Thoughts

MA	Categories	*f*	%	*cf*	*cp*
Appropriate	Quantifying the probability	91	59.09	154	55.39
	Emphasizing probable event	38	24.67		
	Considering other possible factors affecting probability	15	9.74		
	Emphasizing uncertain event	8	5.19		
	Emphasizing the effects of recurrence of trials	2	1.29		
Total		154	100		
Inappropriate	Decision-making based on the consequences of events	35	28.22	124	44.60
	Emotional decision-making	26	20.96		
	Assuming previously observed outcome will repeat	19	15.32		
	Assuming complementary events will occur in sequence	17	13.70		
	Assuming increased probability as certain event	10	8.06		
	Assuming the lower probability as improbable event	7	5.64		
	Assuming the higher probability as certain event	5	4.03		
	Assuming unobserved outcome as improbable event	4	3.22		
	Assuming previously observed outcome as certain event	1	0.80		
Total		124	100	278	100

Note: MA, mathematical appropriateness.

Discussion

In this study, children were expected to predict the possible outcomes of the trials, to make choices against the trials, to express their inferences about the consequences of trials, to make reasonings about complementary events and to express their considerations about repetition of probabilistic trials. Trials consisted of two different wooden wheel mechanisms having black and white circle slices in different ratios, and a progressive game played by the help of wheel mechanisms. Children's responses were categorized under different categories. The categories emerged in this study also reflect children's strategies to understand and explain the probabilistic situations. It was seen that children were trying to compare the probabilities by counting the numbers of circle slices (P2: Black does not seem to come out much. There are one black and three white parts). Children emphasized the concept of probability by considering the chance of both colored circle slices because of at least one circle slice's presenting (P20: Because one of them should have won. Black or white). Children considered other possible factors affecting the spinning of the wheel, as wind coming from the window, spinning too slow or too fast (P22: Windows are wide opened. The wind from the window may have turned it even more). They emphasized the concept of uncertainty because of its being so difficult to predict the outcoming color, until they examine the consequences of trials (P8: We can't know. We need to try and wait for consequences). Even they considered the recurrence of trials effecting the probability (P1: It could come out much if we turned it for more times). Similarly, Way (2003) states that children ratiocinate against the probabilistic situations, they have some understanding about randomness, and they develop some strategies to understand, and to explain the probabilistic situations.

For each mechanism, the children were asked the question, "Which one of these puppies do you think wins the game, if we use this wheel? Why?" This question was asked to elicit children's predictions about the consequences of probabilistic trials. The results show us that most of children's justifications (64.15%) about their predictions were mathematically appropriate. Their predictions were based on "Quantifying the probability," the concept of uncertainty, probable events, and other possible factors affecting probability. Similar to emergence of the category of "Considering other possible factors affecting probability," Lamprianou and Lamprianou (2003) state that primary school students consider the length of the alternative ways as a factor to reach the target, regardless of the possibilities of the ways' to reach the target. In Pratt and Noss's (2002) study, children state that it is not possible to predict which numbered circle slice of the wheel will come out whether it turned slowly or quickly. But in this study, some of the participants' responds were about the effects of the speed of the wheel on probability. For example, P3 stated that "I didn't turn it slowly. Black would always come out if I turned it slowly," and P12 stated that "Black won, may be because I turned it speedily." Similar to this result Van Dooren et al. (2003) state that 10–12-year-old-children have misconception as "assuming that variables in probabilistic situations are linked by a linear relationship."

The children were asked the question, "What color puppy would you choose to win the game? Why?" This question was asked to elicit children's choices about the probabilistic situations. According to the results of this study, most of children's justifications (60.00%) about their choices were mathematically inappropriate. Their choices were mostly based on "Emotional decision-making" (P25: Black will win. Because I like it more). Other underlying justifications of children's choices were based on "Assuming previously observed outcome will repeat" (P18: Black wins. Because it all came out black), "Assuming increased probability as certain event" (P4: I'm sure that black will win now. Because it had only one part before but has two parts now), "Assuming complementary events will occur in sequence" (P24: Black will win now because it couldn't win before), assuming the highest probability as certainty (P17: White comes out every time. Because white is more than black) and "Assuming previously observed outcome as certain event" (P1: Definitely, it will win again. Because it won just before). In this study most of children's justifications related to their choices against probabilistic situations, were found to be as they are mathematically inappropriate. Children thought that the color more times or all times coming out is more likely to happen or a certain event. The expressions of P17 and P1 are examples of "positive recency effect" which is stated in Bryant and Nunes's (2012) study too.

Contrary to these results, Bayless and Schlottmann's (2010) study revealed that 5–7-year-old children may choose mathematically appropriate choices by considering the possibility of winning the game. Falk et al. (2012) studied with 4–11-year-old children. They examined children's inferences about probability by offering binary choices. Although their study shows an increasing rate according to age, it has been revealed that children generally make choices with a high probability of winning regardless of the chance. But in this study children's justifications about their choices for winning the game were out of mathematically appropriate explanations.

The children were asked the question, "What color puppy won the game? Why do you think such a result came out?" This question was for eliciting the children's inferences about the consequences of probabilistic events. As a result of this study most of children's inferences (55.35%) were mathematically appropriate. Their inferences were mostly based *on "*Quantifying the probability" (P2: Black does not seem to come out much. There are one black and three white parts). Others were "Considering other possible factors affecting probability" (P12: Black won, may be because I turned it speedily), "Emphasizing probable event" (P25: Black may come out, white one, too. It may be like that) and "Emphasizing uncertain event" (P16: Whatever I say may be the opposite. It is so hard to guess). Besides considerable ratio of children's inferences (44.65%) were mathematically inappropriate ones. Their mathematically inappropriate inferences mostly based on *"*Decision-making based on the consequences of events" (P3: Both were on the same line. Black came out last, and it won). Others were "Assuming increased probability as certain event" (P23: Now black has multiplied. Of course, it would win), "Emotional decision-making" (P12: Now black will win. Because it was upset that it just couldn't win), "Assuming the higher probability as certain event" (P18: White always come out. Because it has three parts, other has one part). In this study winning the game depends on how many times the color came out. This situation may be the reason of children's mathematically inappropriate responses. "It is easy to predict the possible outcomes of single events for children but, aggregated ones are not*"* (Jones, Langrall and Mooney, as cited in Abrahamson, 2000).

The children were asked the question, "Do you think the other puppy would have won the game, too? Why?" This question was for eliciting the children's reasonings about complementary events. According to the results of this study, most of children's reasonings (60.27%) were found to be mathematically appropriate. Their reasonings about complementary events were based on emphasizing the probability, "Quantifying the probability," "Considering other possible factors affecting probability," and "Emphasizing the effects of recurrence of trials." Similarly, the study by Tatsis et al (2008) reveals that 5-year-old children may overcome their basic intuitions related to the fairness of probabilistic games, by using counting strategy. And Pratt and Noss's (2002) study reveals that children may have judgments about the fairness of a probabilistic game by comparing the amounts of variables. Similarly, P6 stated that "Actually, it could come out, but it is so hard. Because there is only one black part" and P23 stated that "White one wins. Because white and black are different. Black has one but white has three," in this study.

Lastly, the children were asked the question, "What color puppy do you think will win the game if we play it again? Why?" This question was asked to elicit children's considerations on the repetition of probabilistic events. Surprisingly, 50.00% of children's considerations were found to be mathematically appropriate, and 50.00% of them were mathematically inappropriate. Children's mathematically appropriate considerations were based on "Quantifying the probability," "Emphasizing the probability," "Emphasizing uncertain event," and "Considering other possible factors affecting probability." Besides, children's mathematically inappropriate considerations were based on "Assuming complementary events will occur in sequence," "Assuming previously observed outcome will repeat," "Emotional decision-making," "Assuming the higher probability as certain event," and "Assuming unobserved outcome as improbable event." In this study it seems children to infer the consequences of the trials based on personal beliefs or misconceptions, not on data (P13: White wins. Because white won just now. P21: White won just before. Now black will win. P4: I feel that black will be more successful. P7: No, it would not have won. Because white came out for three times, but no black came out). In this study children showed examples of "negative recency effect" as stated in Bryant and Nunes's (2012) study. They ignored the probability of complementary event. Because their decisions were based on their experiences of previous trial. Children saw that the same color came out for more times, sometimes for all times. Therefore, children thought the complementary one is improbable or less likely to happen. They ignored the independence of the probabilities of possible outcomes.

Similar to the results of this study, Kinnear and Clark's (2014) study reveals that 5-year-old children make predictions about the probabilistic situations based on their personal experiences, not based on the data they have gathered. From another perspective, Girotto and Gonzalez (2008) state that 5-year-old children revise their judgments and decisions appropriately, based on the data they have gathered, in repetition of probabilistic events.

In general manner, most of the children's responses (55.39%) were found to be as mathematically appropriate. Besides 44.60% of them were mathematically inappropriate, that is, they carry misconceptions. In this study misconceptions were emerged as they are; "Decision-making based on the consequences of events" (P11: Black came out three times, and it won. But white were behind), "Emotional decision-making" (P24: Isn't it a pity for black, let it win too), "Assuming previously observed outcome will repeat" (P7: White wins. Because white came out when I turned it before), "Assuming complementary events will occur in sequence" (P2: I think white will win. Because they win in turn), "Assuming increased probability as certain event" (P8: I'm sure that black wins. Now black has multiplied. It had one part before), "Assuming the lower probability as improbable event" (P16: No, it can't win. How many times should I tell you? This color has only one part), "Assuming the higher probability as certain event" (P9: Since there is a lot of white, it's certain that white will come out a lot), "Assuming unobserved outcome as improbable event" (P19:

Black can never win. Because white won twice. Black couldn't win), and "Assuming previously observed outcome as certain event" (P16: White just won. Now for sure, white wins).

Similar to the results of this study, Ang and Shahrill (2014) stated that even secondary school students have some misconceptions on probability. Their study reveals that students may have misconceptions such as representativeness, probabilistic judgments based on beliefs, human control is decisive on the sequences of probabilistic trials, and equiprobability bias. Similarly, Jun and Pereira-Mendoza (2002) stated that secondary school students have some misconceptions on probability. They defined them as "probability is unquantifiable," "inferencing probabilistic situations based on the consequences of them," "probability is certainty," "probabilistic judgments based on personal beliefs," "ignoring the correlations on probability," "the recurrence of probabilistic makes predictions difficult," "complementary event will occur by recurrence of trial, for sure," and "developing a method for Quantifying the probability." According to Amir and Scott-Williams' (1999) study, 11–12-year-old children's inference about probability are based on the consequences of events.

Considering the table presented in Appendix 2, which shows the emerging categories and explanations, we may see that children's thoughts about probability are not consistent, in the context of mathematically appropriateness. Of course, many participants demonstrated some mathematically appropriate understandings but many of them have some misconceptions, at the same time. Abrahamson (2009) state that children's inferences about probabilistic situations may be affected by graphical representations. According to the results of this study, children's mathematical appropriate responses may be besed on the visual characteristics of wheels, instead of mathematical calculations. Especially for the category of "Quantifying the probability" (P6: Actually, it could come out, but it is so hard. Because there is only one black part. P12: White one wins. Because it has more places. P23: White one wins. Because white and black are different. Black has one, but white has three). Considering these responses, we may assume that children decided by counting the numbers of circle slices, not by considering the sample space, or the ratios of probability.

We may assume that children's inferences about probabilistic events depends on the conditions of each probabilistic situation (not consistent). For example, P2 and P23 both represented understandings about "Quantifying the probability," and misconception as "Assuming complementary events will occur in sequence." As another example, P4 and P8 both represented understandings about the concept of uncertainty, and misconceptions as "Emotional decision-making" and "Assuming increased probability as certain event." As a final example, P12 and P25 both represented understandings about "Emphasizing probable event," and misconceptions as "Emotional Decision Making." These results are parallel to those of HodnikČadež and Škrbec's (2011) study, where 4–8-year-old children's understandings of the concepts related to probability were examined. Their study reveals that, children's understanding about probability become more consistent, by age. However children may have some mathematical appropriate understandings from the age of four.

Implications

According to the results of this study, children have both mathematically appropriate and inappropriate understandings about probability. Besides some participants responses were not consistent, on their own. On the other hand, some participants' decisions were based on their emotions. In this study puppies as characters of the story and the game were used. The thought of hunger and the color of puppies affected children's responses. Psychologically affective factors such as color, living things may be avoided from the probabilistic further studies with children. Additionally, the underlying psychological factors may be investigated in further research. Some other tools such as flash cards, dice, random generators, or vending machines may be used as alternative tools to bring children's thoughts out. Activities addressed to children's probabilistic thinking may be designed and implemented by using these kinds of tools mentioned earlier.

Ethics Committee Approval: Ethical committee approval was received from the Social and Humanity Sciences Ethics Committee of Ege University (Approval no: 15/10, Date: 28/11/2019).

Informed Consent: Written informed consent was obtained from the parents of children who agreed to take part in the study.

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Appendix 1.
Written Record Form

P	Q1: Which puppy do you think would win if we played the game using this wheel? Why?	Q2: Which one would you choose to win the game? Why?	Q3: Who won the game? Why do you think such a result came out?	Q4: Do you think the other puppy could have won the game too? Why?	Q5: Who do you think would win the game if we played again? Why?
1					
2					
3					
4					
5					
6					
7					
8					
9					

Appendix 2.
Categories and Examples

Categories	**Examples**	**Mathematically Appropriateness**
Quantifying the probability	P2: Black doesn't seem to come out much. There are one black and three white parts. P6: Actually, it could come out, but it is so hard. Because there is only one black part. P12: White one wins. Because it has more places. P23: White one wins. Because white and black are different. Black has one, but white has three.	Children decided, responded, chose, predicted, inferred by quantifying the probability. Although children were focusing on appearance of wheels, their responds were including the amounts or ratios of elements to be occur as sequences of events. (Mathematically appropriate)
Emphasizing probable event	P3: Both can win. Both can come out much. P8: It could win. Because it could come out. P12: Sometimes black comes out, sometimes white. P13: It doesn't matter. White could win if white came out. P14: Both can come out. Both can win. P15: May be black comes out, may be white. P20: Because one of them should have won. Black or white. P25: Black may come out, white, too. It may be like that.	Children's responses included some expressions about the nature of the concept of probability. They considered each element to be likely to come out. (Mathematically appropriate)
Considering other possible factors affecting probability	P2: I turned the wheel with all my power. That's why white came out. P3: I didn't turn it slowly. Black would always come out if I turned it slowly. P10: White won. Because I turned it speedily. P12: Black won, may be because I turned it speedily. P22: Windows are opened. The wind from the window may have turned it even more.	Mechanisms were run by spinning the wheels by hand. That is why children considered the power, speed, and wind which would affect the rotation of wheels. These factors may eliminate randomness. (Mathematically appropriate)
Emphasizing uncertain event	P1: We can't know which one will win. Both can win. P4: We can't know everything. We can't know which color will come out. P8: We can't know. We need to try and wait for consequences. P12: We can't know. We didn't turn it yet. P16: Whatever I say may be the opposite. It is so hard to guess. P18: I don't know. May be white, may be black.	Children's responses were including some expressions about the nature of the concept of uncertainty. They considered obscurity of events and hardness of predicting the events not occurred yet. (Mathematically appropriate)
Emphasizing the effects of recursion of trials	P1: It could come out much if we turned it for more times. P22: White could win if we turned it for more times.	Children considered the wheels' running by hand power and other factors' possibly effecting the rotation of wheels and outcoming colors. (Mathematically appropriate)
Decision-making based on the consequences of events	P2: White came out most. P3: Both were on the same line. Black came out last, and it won. P5: It all came out white. P11: Black came out three times, and it won. But white were behind.	Children focused only on the consequences of events rather than underlying causes of outcomes. (Mathematically inappropriate)
Assuming previously observed outcome will repeat	P4: White wins. Because white came out while I was playing just before. P7: White wins. Because white came out when I turned it before. P13: White wins. Because white won just now. P18: Black wins. Because it all came out black. P24: White wins. Because white won the previous game.	Children considered their previous experiences and observations, and they assumed as they are decisive on the outcomes of events. (Mathematically inappropriate)
Assuming complementary events will occur in sequence	P2: I think white will win. Because they win in turn. P5: White will win. It lost before and will win now. P21: White won just before. Now black will win. P23: White won for one time. Now black will win. P24: Black will win now because it couldn't win before.	Children expressed ideas such as, complementary events will occur in sequence regardless of probability. (Mathematically inappropriate)
Emotional decision-making	P4: I feel that black will be more successful. P8: I'm bored of black. White wins. P12: Now black will win. Because it was upset that it just couldn't win. P24: Isn't it a pity for black, let it win too. P25: Black will win. Because I like it more.	Children expressed their thoughts about probability based on their emotions, such as personal beliefs, admirations, and personal conviction. (Mathematically inappropriate)

(*Continued*)

Appendix 2.
Categories and Examples (Continued)

Categories	**Examples**	**Mathematically Appropriateness**
Assuming increased probability as certain event	P4: I'm sure that black will win now. Because it had only one part before but has two parts now. P8: I'm sure that black wins. Now black has multiplied. It had one part before. P23: Now black has multiplied. Of course, it would win.	Children considered that black one lost the game because of its having only one circle slice against white one. Children ignored the equilibrium and probable event in the new case, that is, both black and white ones having two circle slices. (Mathematically inappropriate)
Assuming the lower probability as improbable event	P7: No, it couldn't have come out. Because black has only one part but white has three parts. P12: No, it doesn't come out. It has one part. P16: No, it can't win. How many times should I tell you? This color has only one part. P22: No, it couldn't have come out. Because whites are three and black is one. P23: It couldn't reach before white one. Because there is very little black.	Children focused on the ratios of circle slices, but they ignored the probable event. Children expressed such an idea that black ones having only one circle slice makes it improbable. (Mathematically inappropriate)
Assuming the higher probability as certain event	P9: Since there are a lot of white, it's certain that white will come out a lot. P17: White comes out every time. Because white is more than black. P18: White always come out. Because it has three parts, other has one part.	Children focused on the ratios of circle slices, but they ignored the probable event. Children expressed such an idea that white one will always win the game because of white ones having more circle slices than other one. (Mathematically inappropriate)
Assuming unobserved outcome as improbable event	P5: No, it wouldn't have won. Because no white came out. P7: No, it wouldn't have won. Because white came out for three times, but no black came out. P19: Black can never win. Because white won twice. Black couldn't win.	Children focused only on the consequences of events. Children expressed such an idea that an event is probable only if it occurs. They ignored the probable event and the concept of sample space. (Mathematically inappropriate)
Assuming previously observed outcome as certain event	P1: Definitely, it will win again. Because it won just before. P5: White always wins. Because every time white one came out. P16: White just won. Now for sure, white wins.	Children focused only on the consequences of events and their previous experiences. Children expressed such an idea that if an event occurred for one time, then it is a certain event. They ignored the probable event and the concept of sample space. (Mathematically inappropriate)
Expressing no idea	P14: Don't know. P15: I don't know P18: Actually, I don't know.	Children expressed no idea.