İSTANBUL UNIVERSITY CERRAHPAŞA

HAYEF: Journal of Education, 21(3); 232-243

DOI: 10.5152/hayef.2024.23093

Received: October 24, 2023
Revision Requested: November 15, 2023
Last Revision Received: November 16, 2023
Accepted: November 17, 2023
Publication Date: September 19, 2024

HAYEF: Journal of Education

RESEARCH ARTICLE

The Views of Teachers with Project Experience on Early Mathematics

Feyza AYDIN BÖLÜKBAŞ[1], İlkay ULUTAŞ[2]

[1]Department of Basic Education, Department of Preschool Education, Aksaray University, Faculty School, Aksaray, Türkiye
[2]Department of Basic Education, Department of Preschool Education, Gazi University, Faculty Education, Ankara, Türkiye

Abstract

This study included 15 pre-school teachers with experience in mathematics-based projects to examine their views on early mathematics. The data were collected through semi-structured interviews with the teachers and analyzed by content analysis method. As a result of the study, the teachers stated that they were primarily based on the cognitive domain acquisitions and basic mathematical skills emphasized in the Ministry of National Education pre-school education program and that children had difficulty with three-dimensional shapes, time, reasoning-association, graphing, measurement, and right–left discrimination. In addition, while organizing the early mathematics education process, it was determined that they paid attention to being in contact with families, thinking that mathematics is a whole, ensuring that children learn from concrete to abstract and notice the mathematics in their environment, preferring methods such as observation, evaluation according to achievements, conversations/dialogues and asking questions to evaluate individual differences, and trying to reduce individual differences with individual support, cooperation with family, and peer learning. It was determined that the mathematics projects provide positive reactions from families, sharing information with colleagues and academicians, and conducting research, and that they made suggestions such as in-service training, being open to learning, family involvement, qualified undergraduate education, mathematics activity collection, and e-Twinning projects for quality early mathematics education.

Keywords: Mathematics education in early childhood, mathematics projects, pre-school teacher, quality early math education, teacher in early mathematics

Introduction

From an early age, children try to recognize and understand concepts related to mathematics and make predictions (Donlan, 2020). Prior literature shows that math-related awareness is learned at an early age, which helps to develop numerical processing ability and can have a positive impact on school performance in the future (Björklund et al., 2020; Clements & Sarama, 2020; Donlan, 2020; Duncan et al., 2007). Therefore, although the importance of mathematics teaching and learning in the early years is accepted unquestionably (Tzekaki, 2020), its abstract and dynamic structure raises questions about how mathematics education can be conducted effectively (Björklund et al., 2020).

Regulating the process of learning and teaching mathematics in early childhood education requires more comprehensive and in-depth processes compared to later school levels. Everyone connected with qualified mathematics education for children should develop children's mathematical interests, build on their families and children's experiences, plan and apply mathematics curriculum and teaching practices in accordance with children's cognitive, linguistic, physical, and social-emotional development, use curriculum and teaching practices that strengthen problem-solving and reasoning processes, integrate mathematics and other activities, provide adequate time, materials, and teacher support to children, actively introduce mathematical concepts through appropriate experiences and teaching strategies, and support all children's learning by evaluating their mathematical knowledge, skills, and strategies (NAEYC (National Association for the Education of Young Children) & NCTM (National Council of Teachers of Mathematics), 2010). If this process is not planned well, as the school level progresses, it may become more difficult for children to close the gap between concrete practical thinking and abstract logical thinking and to catch up with the mathematical levels of their peers (Bodovski & Farkas, 2007; van Oers & Poland, 2012). At this point, it is becoming a more important issue to create qualified learning experiences and environments that are appropriate for the individual differences of children in early mathematics skills (Hardy & Hemmeter, 2019). In addition, it is stated that the organization of high-quality early mathematics programs is especially effective in improving the mathematical skills of children who are behind their peers. It is essential to identify children with insufficient math skills or who are at risk of falling behind and conduct appropriate assessments to provide them with the necessary early math environments. Developmentally appropriate intervention programs for early mathematics education increase children's mathematical knowledge and improve their mathematical performance (Yazıcıoğlu & Akdal, 2023).

Even if adequate mathematical opportunities and environments are provided, teachers act as a bridge for children to make mathematical connections (Clements & Sarama, 2020; Sprenger & Benz, 2020). This role requires the teacher to plan the learning process, teaching methods, and environments in a manner that improves children's mathematical learning ways and thinking skills (Björklund et al., 2020; NCTM, 2013). Planned in accordance with the individual needs of children, designed to enable them to develop their existing skills and learn new skills, arouse their curiosity and desire to explore, be developmentally appropriate, culturally and linguistically sensitive, and comprehensively

Corresponding Author: Feyza AYDIN BÖLÜKBAŞ, **E-mail:** feyzaaydin@aksaray.edu.tr

Cite this article as: Aydın Bölükbaş, F., & Ulutaş, İ. (2024). Examining the views of teachers with project experience on early mathematics. *HAYEF: Journal of Education*, 21(3), 232-243.

organized in such a way as to support all areas of development, early mathematics programs are more effective in children (NAECS/SDE (National Association of Early Childhood Special-State Departments of Education), 2003). Consistent and engaging mathematics education in a way that responds to developmental needs allows children to love mathematics, continue to explore it, and relate it to their daily experiences. Thanks to this supportive structure, even children having a bumpy ride have the opportunity to participate in mathematical interaction and express their mathematical ideas (Breive, 2019; Vogler, 2019).

To support high-quality early mathematics education, organizations and program developers need to create more effective early childhood teacher training and professional development; use collaborative processes to develop systems with appropriate high-quality standards, curricula, and assessment; design organizational structures and policies that support teachers' continuous learning, teamwork, and planning; and provide the necessary resources to overcome barriers to young children's mathematics competence in the classroom, community, institutional, and system-wide (Gasteiger & Benz, 2018).

Çerezci (2019) concluded that there are significant differences in the quality of teachers' mathematics teaching. While defining teachers who deepen children's mathematical understanding as higher quality, she also defined teachers who provide superficial education by focusing on methods and content as low quality. Teachers' participation in professional development training programs is reflected in their knowledge, beliefs, and attitudes (Fishman et al., 2003). It is seen that professional development training programs are essential when teachers are required to update their practices and to identify areas where they may be lacking in the education they provide in the classroom. It is crucial to support them in experiencing new strategies in the programs they attend for their professional development, to observe them with examples in the real classroom environment, and to support them in incorporating these strategies into their practices to create quality early mathematics (Chen & McCray, 2012).

When the studies involving teachers' views on early mathematics in Turkey are analyzed, it is revealed that they are related to gender differences (Erden & Tonga, 2020), mathematics activities and classroom applications (Karakuş et al., 2022), giving a place to NCTM content and process standards in their activities (Pekince & Avcı, 2016), planning and implementing math activities (Kaçan & Halmatov, 2017), assessment and family participation dimensions (Akıncı-Coşgun & Yılmaz, 2021). In addition, it was indicated that most early mathematics studies were conducted with children, and quantitative research methods were used (Altan et al., 2021). However, there is a dearth of research on the experiences of teachers who implement project-based mathematics applications in their classrooms.

Since early mathematical skills form the basis for later mathematical skills, the Mathematics Mobilization initiated by the Ministry of National Education and the e-Twinning, Scientix, Future Classroom Lab, DesignFILS, Novigado and Edusimsteam projects carried out under the coordination of the General Directorate of Innovation and Educational Technologies (YEĞİTEK) have come to the fore in our country based on the importance of providing quality content and environments to children in the pre-school period (https://matematik.eba.gov.tr). In addition, a special call was opened for mathematics in the 2022–2023 period in TÜBİTAK Science and Society-Based Projects. Many pre-school teachers carry out math-based projects in their classrooms by taking advantage of these resources and collaborations, and go on the way to enriching their experiences theoretically and practically by taking academic support. Examining the experiences of teachers who attach importance to mathematics education can be a guide for other teachers and researchers. This study was conducted in order to determine the experiences of pre-school teachers who have project experience in mathematics. This research will seek answers to the following questions:

- What are the concepts/skills that pre-school teachers consider important in early mathematics education?
- What are the ways for pre-school teachers to evaluate children's math achievements and reduce individual differences?
- What are the achievements of pre-school teachers from mathematics projects?
- What are the recommendations of pre-school teachers about high-quality mathematics education?

Methods

This study is structured with phenomenology, one of the qualitative research methods. According to Creswell (2023), phenomenology research aims to make deep sense of individuals' experiences and perceptions by enabling participants to describe and detail their experiences and to reveal common structures. In the research, phenomenology has provided an examination of the early mathematics-related experiences of teachers who have been involved in various mathematics-related projects.

Study Group

The study group of the research consisted of pre-school teachers working in pre-school education institutions in the 2022–2023 academic year. In order to determine the teachers in the study group, the teachers were determined primarily by the criterion sampling technique, which is a type of purposeful sampling method. Participants who meet certain criteria in the criterion sampling method are included in the study (Büyüköztürk et al., 2017). When determining the teachers to be included in the study group, it was taken into account that the teachers had taken part in early mathematics-related projects.

Accordingly, pre-school teachers who participated in e-Twinning and TÜBİTAK projects involving mathematics were contacted through social media, and 15 pre-school teachers volunteered to participate in the study. The demographic characteristics of the pre-school teachers participating in the study are given in Table 1.

Considering the demographic characteristics of the pre-school teachers participating in the study, it was found that the majority (f = 8) were between the ages of 30 and 35 and had a bachelor's degree (f = 10). When the professional experience of the teachers is analyzed, it is seen that most of them (f = 8) have 11–15 years of experience and the other teachers (f = 3) have 6–10 years and 16–20 years (f = 4) of professional experience. When the provinces where they worked were analyzed, it was determined that they worked in five different provinces (Ankara, Erzincan, Balıkesir, Kocaeli, Edirne), and most (f = 10) worked in Ankara. Most teachers (f = 10) participated in TÜBİTAK and e-Twining projects (f=8). They include mathematics education (every day f = 13) in their classes. The factors that enabled them to realize the importance of early mathematics were undergraduate education (f = 6), projects (f = 6), and the trainings they attended (f = 5).

Data Collection Tools and Data Collection Process

In order to collect the data in the study, first of all, the literature was examined, and the "Pre-School Teachers" Views on Mathematics Education Determination Form was created. The form was submitted to five experts in the field of pre-school education and evaluated in terms of scope and comprehensibility, and after the corrections were made according to the expert feedback, an Ethical Permission application was made. After the approval was obtained from Aksaray University Human Research Ethics Committee, a pre-experiment was conducted by interviewing a teacher (Approval no: 2023/03-71, Date: 25.04.2023).

Table 1.
Demographic Information of Teachers

Demographic Characteristics			*f*
Age	30-35 years old		8
	36-40 years old		3
	Over 41 years old		4
Educational status	Undergraduate		10
	Master		5
Professional experience	6-10 years		3
	11-15 years		8
	16-20 years		4
The province where they work	Ankara		10
	Erzincan		2
	Balıkesir		1
	Kocaeli		1
	Edirne		1
Projects they participated in related to early mathematics	TÜBİTAK project		10
	E-Twinning project		8
	In-service training		5
	Mathematics mobilization workshop		3
	Montessori education		1
Resources related to mathematics education	Social media etc.		15
	Academic publications		2
	Children's books		3
	Activity books		2
Frequency for inclusion of mathematics education in the program	Everyday	15-20 minutes	1
		30 minutes	1
		1 hour	5
		At every opportunity	6
	3 days a week	1 hour	1
	4 days a week	30-45 minutes	1
Factors that enable them to realize the importance of early mathematics	Undergraduate education		6
	The trainings they attended		5
	Projects (E-Twinning, TÜBİTAK)		6
	Following academic publications		2

With the experiences obtained from the preliminary trial, the form was finalized, and the actual application was started. For the actual application, teachers who took part in early mathematics-related projects were contacted via social media, and information about the study was provided. "Informed Consent Form" was presented to the teachers included in the study, and necessary information about the research was given. Since the teachers work in different provinces, interviews were conducted with online sessions. The interviews were audio recorded with the approval of the teachers at the beginning of the interviews, and the interview forms were filled in after the interviews were completed.

Data Analysis

The data obtained from the interviews with pre-school teachers were analyzed by content analysis. The main purpose of content analysis is to reach concepts and relationships that can explain the collected data. The findings are interpreted by bringing together similar data within the framework of certain concepts and themes (Creswell, 2023; Yıldırım & Şimşek, 2016).

In the research, the audio interview recordings obtained from the teachers were transferred to a computer environment and transcribed. Codes such as T1, T2, T3 are given within the scope of ethical principles to be used in direct quotations from teachers. The general structure and content of the data obtained from the teacher interviews were analyzed to understand the related structure. Then, codes were created to divide the obtained data into categories. A detailed examination was made during the creation of the coding system, and after completion, similar codes were combined, and common themes and subcategories were revealed. For the explanation and interpretation of the themes, the data were analyzed in detail and presented together with teacher quotations.

Validity and reliability in qualitative research should be considered in different contexts than quantitative research. The precautions taken and strategies applied for validity and reliability in the research are explained. In qualitative research, validity criteria such as "expert opinion, participant confirmation, inter-coder reliability, and rich description" are recommended to look holistically and to verify the consistency of the results. In the study, expert opinion, participant confirmation, and inter-coder reliability were utilized to ensure internal validity. In this context, expert opinion was taken while formulating the interview questions, no change was made in the number of questions, which was nine, and adjustments were made in the wording of the questions. Feedback from participants is required in data and ambiguous interpretations. In the current study, feedback was received from teachers for participant confirmation during and after data collection (Merriam, 2015; Yıldırım & Şimşek, 2016). When the inter-coder agreement was calculated (Agreement/(Agreement+Disagreement) × 100), the agreement rate was 87.5%. Since the inter-coder reliability coefficient is above 80%, it shows that the reliability is sufficient (Miles & Huberman, 1994). Detailed descriptions and purposeful sampling methods are proposed to ensure external validity in qualitative research. The opinions of the participants should be described in detail at this point, in order to ensure depth and integrity with rich descriptions, the opinions of the teachers are presented in detail in the research findings with direct quotations. Purposeful sampling was also used when determining the sample group of the research. External validity provides transferability; thanks to these methods, it supports being more experienced and aware when research is transferred to similar environments and processes (Merriam, 2015; Yıldırım & Şimşek, 2016).

Reliability is explained separately as internal reliability and external reliability. Considering that researchers perceive and interpret situations and events differently at the point of internal reliability in qualitative research, evaluating it as scientific and taking measures for it is recommended. In the current research, among the strategies suggested by LeCompte & Goetz (1982) regarding internal reliability, direct presentation of the collected data and the involvement of more than one researcher in the research were used to ensure internal reliability (Yıldırım & Şimşek, 2016). Detailed information about all stages of research related to external reliability should be included and credibility should be increased. In the study, raw data were analyzed by two different researchers at different times and efforts were made to prevent personal interpretations that may occur in the results. In addition, sample selection and interview process were explained in detail (Yıldırım & Şimşek, 2016).

Results

In line with the content analysis of the interviews with the teachers, seven themes were obtained as *"concepts/skills addressed in early mathematics education", "mathematical skills that children have difficulty with", "points to be considered in the mathematics education process", "ways to reduce individual differences", "evaluation of mathematics achievements", " achievements obtained in the mathematics education process" and "suggestions for quality early mathematics education"* (Figure 1).

Each theme is discussed separately under the heading and explained below.

Concepts/Skills Addressed in Early Mathematics Education

Examining the opinions of the teachers about the concepts they addressed in early mathematics education, it was found that the majority

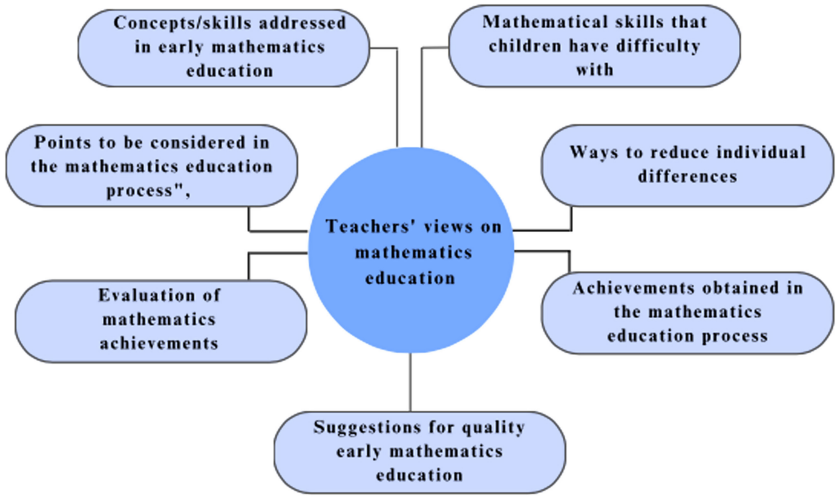

Figure 1.
The themes involving the views of teachers with project experience on mathematics education.

of them (f = 9) reported relying on the cognitive domain acquisitions and basic mathematical skills emphasized in the Ministry of National Education pre-school education program. In addition, some teachers stated that they gave more space to meaningful counting (f = 2), pattern (f = 2), problem-solving skills (f = 2), and cause-effect relationship (f = 1) skills in practice (Figure 2).

Regarding the cognitive domain acquisitions and basic mathematical skills, T7 expressed his/her opinion as follows.

T7: "We have all of them. There is no specific concept you elaborate. At first we started with a comparison. We continued with the ranking. We continued with pairing. We have progressed step by step."

Stating that he/she focuses on the cause–effect relationship and problem solving, T3 stated his opinion as follows.

T3: "I make them focus more on cause-effect relationships. In other words, when the child sees the cause or the result of something, it is easier for me to reach a concept or it helps me to find the way I will go or the way he/she will find after reaching that concept."

T9's opinion about meaningful counting is as follows

T9: "for example, let's say I gave the child five objects. When I want these five objects, I want them to be complete, or when I show him/her the number five against him/her, when I want him/her to complete it with the object, I want him/her to select and complete it, let's say five toys from there."

The views of T2, who stated that the pattern was important in early mathematics:

T2: "I think there is a little confusion in the pattern. For example, they can rank, group very quickly. They can distinguish quickly. But I'm thinking about patterns. And when I gave the first pattern, they always thought it would only be wrapped in yellow, red, yellow, red. Now, I think that children's creativity develops in such a way that every time they say, "oh, there are also seasons, there are also days of the week."

The Points Where Children Have Difficulty in Early Mathematics

Teachers stated that children had difficulties in three-dimensional shapes (f = 4), time (f = 4), reasoning-association (f = 3), graphics (f = 3), meaningful counting (f = 3), recognizing numbers (f = 2), measurement (f = 2), and distinguishing right and left (f = 1) (Figure 3).

When the teacher statements were analyzed, many difficulties were expressed such as

T3: "if I think about the beginning of learning, children have difficulty in measurements. They have difficulty reading graphics. For example, the concept of right-left that I have observed, I mean, they learn it hard, but then it seems getting easier. I also

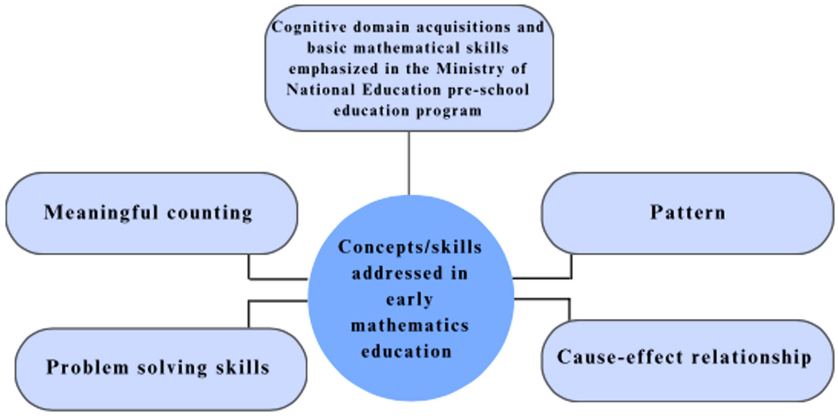

Figure 2.
Concepts/skills addressed in early mathematics education.

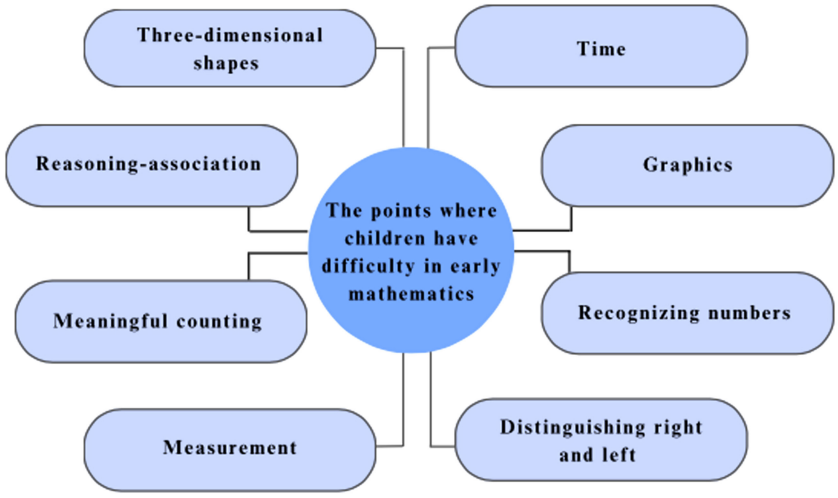

Figure 3.
The points where children have difficulty in early mathematics.

observe that they have difficulty with time. So, for example, the concepts of yesterday and tomorrow are a little bit difficult to grasp. For example, they confuse the concepts of broad and narrow."

T4 stressed the following while explaining three-dimensional shapes:

T4: " for example, when they see a circle in a picture, they see a half circle, they do not say half, they call it a circle directly, or if they see a cube, they call it a square, if they see a sphere, they call it a circle. For example, they cannot do these"

T8 regarding the concept of time:

T8: "Today, tomorrow, they all mixed together. For example, they say tomorrow, my teacher, I came to you, they say, for example. They say I was coming to you tomorrow. Then they have a lot of shortcomings in the concept of time."

It is observed that children have difficulties, especially in the concept of time and spatial perception skills that require mathematical or spatial thinking.

Points of Attention When Organizing the Mathematics Education Process

While organizing the education process related to early mathematics, the teachers stated that they paid attention to points such as being in contact with families ($f = 7$), thinking of mathematics as a whole ($f = 6$), learning from concrete to abstract ($f = 6$), enabling them to notice the mathematics around them ($f = 6$), progressing from the concepts they see missing ($f = 4$), gaining a love for mathematics ($f = 4$), preparing activities for their developmental level and previous knowledge ($f = 3$), and using free time effectively ($f = 2$) (Figure 4).

T3's views on family are as follows;

T3: "what I have noticed in these last five years and what I am trying to implement is to create awareness in the child and in his family. For example, we call it literacy preparation study or maths study. The first thing that comes to their minds when they hear the word "maths" is the number or learning the numbers. From my point of view, I made changes in the direction of creating awareness about these issues for parents and then students that math activities are not just about this."

T4 emphasizing the points of learning, having fun, loving mathematics, and relating it to life, expressed:

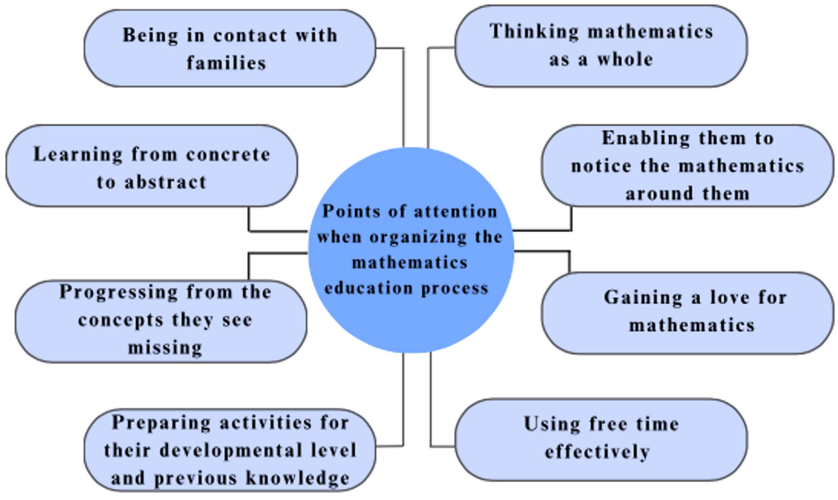

Figure 4.
Points of attention when organizing the mathematics education process.

I say at first that comparison is fundamental at all events. Then I make them have fun, love it. Because if you don't make it fun, there is something mutually negative in the perception of mathematics in children, so I want it to be experienced fun in practice. I want them to realize that mathematics is actually something that is not very difficult. So the preliminary basic point is to have fun, attract their interest, and when you combine this with different types of activities, they already see it. Associating with life is actually the important point.

In addition, T12 also stated that:

For example, on a small rectangular board, screwing a nut, but the sizes are different, and the children make the concept of big and small and which one goes with which one completely by looking inside the nut and using visual perception. Additionally, I'm using keys and locks. So I'm trying to attract the attention of children by creating more centers like this. We already have a math center

Stating that it enables children to understand mathematics by presenting materials from the objects around them.

Ways to Reduce Children's Individual Differences in Mathematics

Teachers stated that they tried to reduce the individual differences of children in mathematics by using individual attention (f = 13), cooperation with family (f = 6) and peer learning (f = 4) (Figure 5).

T4 expressed his/her opinion about individual attention as follows:

Whatever they ask me, I will definitely answer, I will definitely listen. The process of their talking to me about mathematics is effective there. It happened even today. Let me say it was a little more difficult; I provide close support.

Considering the views of T8 on this issue:

T8: "*I don't just think about it mathematically. For example, language development is lagging behind. For example, they are behind in the social sensory field, behind in mathematics. There is such slight differences in all of them. Sometimes there is individual and sometimes peer teaching, I take those who can and those who cannot at the same table, sometimes they do it quickly."*

It was found out that it provides individual attention and peer support to children not only in mathematics development but also by taking into account other areas of development.

In order to reduce individual differences, T2 stated that it uses individual and peer support in the following way:

T2*: " I'm trying to make it a little more difficult for the children who advanced more. I also try to take them individually. The best time for us is free time. In my free time, I take the children who advanced more and the children who are behind individually. Then I integrate them. I make sure that the student who is at an advanced level and the student who is at a slightly backward level play games together, of course, we always teach with games. I make sure that they interact together during the game. Because they really like to help each other."*

T12 stated using peer and family support as follows:

T12: *"For example, I do peer education by planning a child who needs support to sit between two children in the activity. Apart from that, I send some activities to families at certain intervals. Games they can play or responsibilities at home, usually kitchen skills, household responsibilities."*

Evaluation of Mathematics Achievements

When the teachers' opinions about the ways of assessing early mathematics outcomes are analyzed, it was revealed that they frequently reported that they made observations (f = 10). In addition, it was determined that they also used ways such as evaluation according to the outcomes (f = 4), engaging in mathematics conversations (dialogues) (f = 2) and asking questions (f = 2) (Figure 6).

Stating that he/she made observations and conversations in order to evaluate the achievements of the children, T4 stated his/her opinion as follows:

T4: "I am constantly observing. I'm asking, what is your idea. I try to involve them in the conversation as much as possible. If they disagree, I do not force them at that moment, then I ask them individually. What did you do? Did you get it? What part of it didn't you understand? I support them in that sense. When it is a year, the achievement is ahead of the indicator, the children are aware that they are getting it. They have awareness of measurement. They have an awareness of shape, geometry. I noticed on the math talk..."

T3 stated *"I always sit the children down and make an end-of-day session. Sometimes if I don't do it at the end of the day, I can also apply the end of the day after the event. For example,*

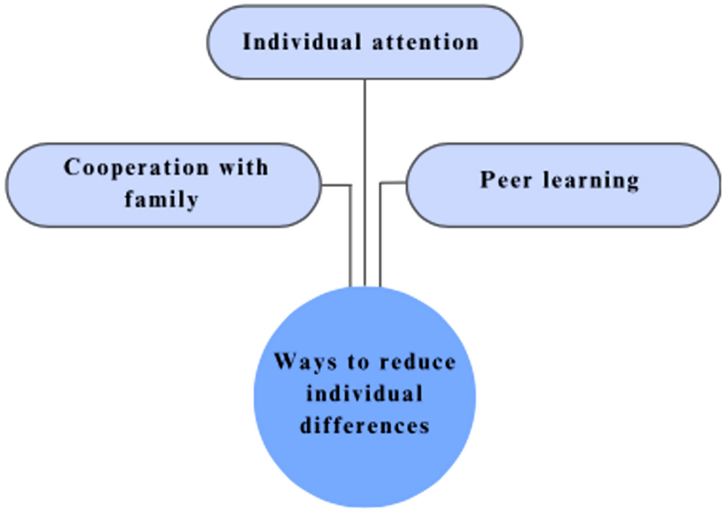

Figure 5.
Ways to reduce individual differences.

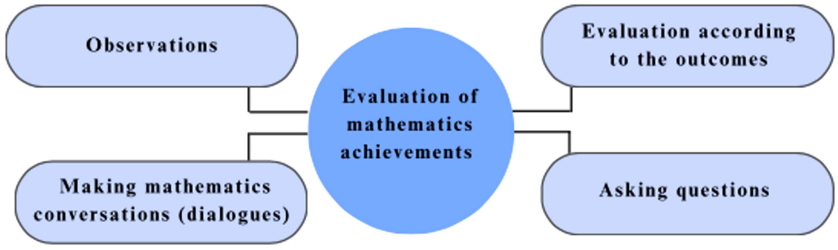

Figure 6.
Ways to evaluate children's achievements in mathematics education.

we played a game of grouping according to colors. During the conversation, I ask the child who had difficulty there, what did you have difficulty with? Were the reds hard to find? Were the yellows found difficult? Which one did you have difficulty in? I do my evaluation by making them talk with questions like that."

He/she stated that he/she tried to reveal the achievements of the children with questions.

Achievements Obtained in the Process of Mathematics Projects

During the early mathematics education projects, it was found that the teachers gained positive reactions of families ($f = 5$), sharing information with colleagues ($f = 2$), sharing information with academicians ($f = 2$) and conducting research ($f = 2$) (Figure 7).

Ö2 expressed his/her views on sharing information with academics, conducting research and the positive reactions of families as follows:

the support of academics in mathematics education projects was very important for my development. I do a lot of research before making a plan. I'm examining different activities. The reaction of the families is also good. I'm getting positive feedback. Parents say that their math skills have improved. They say their child is beginning to be more aware...

Confirming the accuracy. Research. Yes, I would say this is how it is applied. If what I'm doing is wrong, I'll fix it. Making decisions together. Yes, it can force my colleagues. Because projects are orientated towards teaching. When they go on the same path, it brings great success. It's a super improvement.

Ö6 stated the following about the positive reactions of the families:

parents always say when they go to first grade and second grade that teacher, you forced us, but it's good that you forced us. Because we have seen a lot of benefits. I mean, I've been working at the same school for twelve years, I get feedback like this. Every year...

by drawing attention to the fact that it facilitates adaptation during the transition to primary school.

Recommendations for High-Quality Early Mathematics Education

When the opinions of the teachers about what should be done for quality early mathematics were examined, they made suggestions such as in-service training ($f = 7$), being open to learning ($f = 4$), family involvement ($f = 3$), high-quality undergraduate education ($f = 2$), mathematics activity collection ($f = 2$), making mathematics projects ($f = 1$), using technology effectively ($f = 1$), following children individually and providing appropriate support ($f = 1$), using natural materials ($f = 1$), finding different techniques ($f = 2$) (Figure 8).

On the topic of in-service training:

T4: *"Teacher training. Because there should be correct practice and practical training. I think this point will end with teacher training. They need to learn how unnoticed mathematics is..."*

and emphasized that the role of teachers is more decisive in mathematics education.

T1 emphasized that "*...I think it is necessary to have more natural materials, materials that children can sit individually and take into their hands and be interested in. It is necessary to benefit from all kinds of children's wishes and readiness. Of course, I also think that the teacher should be willing in this regard, so that training can be given on this subject*."

and drew attention to natural and concrete materials, readiness and teacher training.

When the views of T12 on the importance of qualified undergraduate education are examined;

T12: *"Education in universities is very important. For example, my colleagues do not know how to write math activities. I mean, when I say mathematics activity, it is perceived as adding, making a pattern. Besides, they do not know how to gamify. I mean, they really don't know that mathematics can be taught through games. In this sense, this should definitely be supported by education at universities. Secondly, there are quality written plans. For example, in that period, we wrote very high quality plans in the teaching practice course. Very good games, materials, these plans*

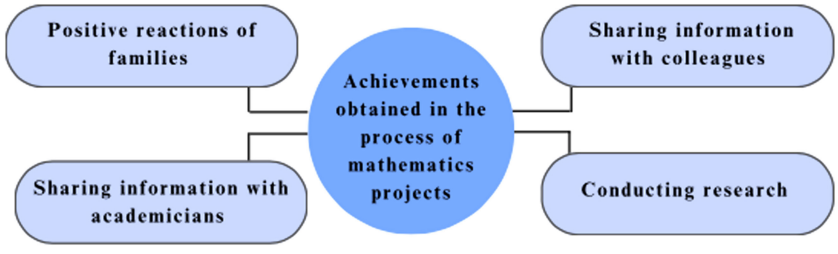

Figure 7.
Achievements obtained in the process of mathematics projects.

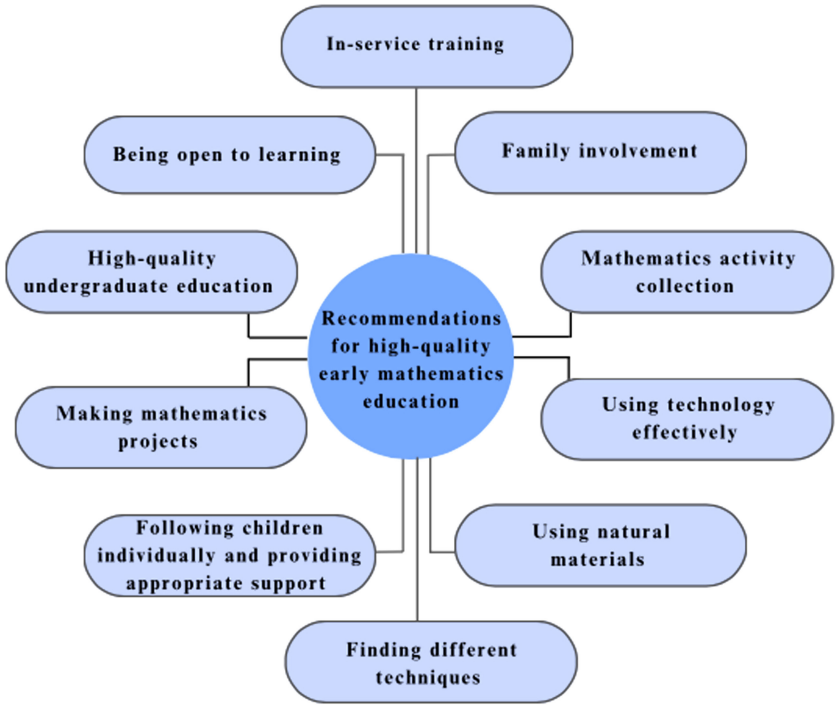

Figure 8.
Recommendations for high-quality early mathematics education.

can be collected and actually something can be done. Under the name of exemplary mathematics activities, there can be activities that can provide my colleagues with such predictions, that is, activities that can be an example, that can offer ideas."

It is seen that the content of the undergraduate program in terms of mathematics education, its integration with games and activity planning is clearly defined.

For quality early mathematics education,

T13: "*I mean, mathematics needs to be repeated in many different ways. I mean, for example, if you just give it on paper without writing numbers, it is not enough. It is necessary to include concrete objects a lot, and you can do it in many different ways, with different methods, for example, you can do it with a digital story, then you can give the same concept again with a concrete material. Then you can give it again on paper. In other words, I think it is necessary to re-present it again and again using different techniques in order for it to be qualified. I think that a single way is not enough for children.*"

Emphasized that it is important to use different materials and methods and that it should not be presented only with paper or a single material.

Discussion, Conclusion, and Recommendations

This study, which examined the views of pre-school teachers on quality early mathematics education, revealed themes such as concepts/skills considered important in early mathematics education, mathematics skills that children have difficulty with, points to be considered in the mathematics education process, ways to reduce children's individual differences in mathematics, ways to evaluate mathematics gains, gains obtained during mathematics projects, and suggestions for quality early mathematics education.

When the opinions of teachers about the concepts they cover in early mathematics education were examined, it was determined that the vast majority of them are based on the cognitive domain achievements and basic mathematics skills emphasized in the Ministry of Education Pre-school Education Program, while some teachers give more space to meaningful counting, pattern, problem-solving skills, and cause-effect relationship. Although teachers take basic achievements and skills as guides, they can place some skills more often according to children's interests, needs, professional experience, and interests. When we look at the previous studies, it is noteworthy that teachers focus on number and counting skills (Baki & Karadeniz Hacısalihoğlu, 2013; Fırat & Dinçer, 2018; Karakuş et al., 2022; Orçan Kaçan & Halmatov, 2017; Yazlık & Öngören, 2018). Although mathematics education is based on cognitive development, it has a rich coverage with content (counting, algebra, geometry, measurement, data analysis, probability) and process (problem-solving, reasoning and proof, communication, association, representation) standards. In this direction, it is noteworthy that the teachers participating in the research give place to the content and process, but their skills for the process are less indicated. In order to provide comprehensive mathematical learning and support children's learning experiences, it is important that teachers also use pedagogical knowledge, and mathematical content and processes effectively (Björklund & Barendregt, 2016). Accordingly, Lee and Ginsburg (2009) stated that teachers should think broadly in early mathematics education and emphasized the deep and comprehensive support of mathematics skills. In this context, problem-solving, graphics making, data analysis, and communication skills that support the mathematics content should also be included more in the process.

The findings obtained from the teachers revealed that children have difficulty in skills such as three-dimensional shapes, time, reasoning-association, creating graphics, meaningful counting, recognizing numbers, measuring and distinguishing right and left. In the study of Orçan Kaçan & Halmatov (2017), teachers stated that children have difficulty with numbers, processing, and measurement. Abstract thinking in mathematics is considered as a process in which children reorganize pre-determined mathematical symbols and structures (Tzekaki & Papadopoulou, 2017). Content and processes such as measurement, time, graphics, recognizing numbers, and distinguishing

right and left require children to reorganize mathematical symbols and certain structures. In high-quality early mathematics practices, it is important to use strategies such as making children aware of mathematical content and situations in their environment, involving children in the learning process in a consistent and level-appropriate way, creating enriched educational environments that include different methods or alternative practices based on their daily experiences (Vogler, 2019; Tzekaki, 2020), filling in the concepts for them to think deeply, and making their learning concrete with the use of materials (Lee & Ginsburg, 2009). However, the math learning process changes with the child's development, readiness, and experiences. Every child can get stuck or jump to different points in their learning process. The difficulties encountered can be facilitated by child-specific learning paths and mathematical content that moves from the concrete to the abstract.

When the points that the teachers paid attention to while organizing the mathematics education process were examined, the following findings were obtained: being in contact with families, thinking mathematics as a whole, learning from concrete to abstract, enabling them to notice the mathematics in their environment, progressing from the concepts they found missing, gaining a love for mathematics, preparing activities for their developmental level and previous knowledge, and using free time effectively. Since the cooperation of family and teacher in the mathematics process ensures that early mathematics education is supported at home and the environment and opportunities offered to the child are increased (Clements & Sarama, 2020), it ensures that families are self-confident about mathematics (Sonnenschein et al., 2021) and reduces the inequalities arising from the socioeconomic level (Ersan & Rodriguez, 2020) and therefore increases the child's school readiness (Lombardi & Dearing, 2021). For this reason, it is necessary to cooperate with the family in mathematics education.

It has been found that the teachers participating in the study evaluated mathematics as a whole. Teachers may have evaluated mathematics as a whole in terms of conducting project-oriented activities. Erden & Tonga (2020) stated in their study that children's early mathematics skills should be supported with activities that are connected and integrated with their daily life experiences. Mathematics is an interdisciplinary field related to psychology, education, and neuroscience (Gilmore et al., 2018). Mathematics needs to progress integrally in terms of its structure, and it is recommended that concepts be given integrally rather than given as separate skills. However, within this integrated structure, the areas or methods to be integrated with the mathematical skills to be acquired by children should be coordinated in a balanced manner (Lee & Ginsburg, 2009). In order for the child to see mathematics as a whole, useful activities would be those that they can associate with life, where they will see mathematics as an important part of life, rather than transaction-based problems.

It has been determined that pre-school teachers also emphasize the concrete-abstract balance in the points they pay attention to in early mathematics education. In the study of Yazlık & Öngören (2018), teachers stated that they both benefited from concrete materials and used number cards by paying attention to the principle of transition from concrete to abstract in early mathematics. Lee & Ginsburg (2009) examined teachers' misconceptions about early mathematics and stated that one of the misconceptions was working only with concrete materials. They emphasized that challenging and encouraging mathematics content should be presented to children and that mathematical learning should not be limited to the concrete one. It can be said that the teachers participating in the study should pay attention to the concrete-abstract balance, that they are aware of the progress from concrete to abstract in learning, and this will make an important contribution to supporting children's reasoning processes.

When looking at how teachers evaluate early mathematics skills, it has been found that they are in the form of observation, assessment according to achievements, conversations/dialogues, and asking questions. Accordingly, evaluations should be made to understand whether mathematics education processes meet the needs of every child (NCTM, 2013). It is emphasized that teachers should use observation, documentation, and other evaluation tools in early childhood education assessments in a way appropriate for developmental, cultural, ethical, ability, and language and in cooperation with other adults around the child. The main purpose of the assessments is to decide how the learning processes should be organized in the most appropriate way for children (Clements & Sarama, 2020; NAEYC, 2020). Ceylan & Aslan (2023) suggest that individual differences in early mathematics skills appear from preschool, so assessments should be made with child-centered measurement tools. Anthony et al., (2015) emphasize that teachers evaluate children only during math-related activities, but studying children thoroughly during the educational process is important. They also state that making the assessments qualified is important in terms of increasing the potential of early math opportunities. Akıncı-Coşgun & Yılmaz (2021), stated that teachers did it after the activity in early mathematics, but they did not use evaluation methods such as portfolio, observation, interview, anecdotal record, etc., and that they lacked knowledge about evaluation. At this point, although it is important for preschool teachers to use different methods when evaluating math skills, it can be said that the evaluation will provide clearer data about children when it is performed in a multifaceted and as a whole.

It was determined that teachers try to reduce the individual differences of children in mathematics education by individual attention, cooperation with the family, and peer learning. Mathematics knowledge begins to be acquired from an early age and forms the basis for more complex mathematical thoughts and academic skills in later periods (Björklund et al., 2020; Donlan, 2020; van Oers & Poland, 2012). It is quite difficult to eliminate the gaps in children's learning in the later periods (Duncan et al., 2007; Reikerås & Salomonsen, 2019). Çakır & Ergül (2022) reported that 42% of children with low early mathematics performance had low working memory performance and had higher risk levels for future mathematical failures. Moreover, Koç (2017) stated that individual differences caused by differences in children's math learning speeds and excessive classroom availability are the points where teachers have difficulty. One of the most important points in individual differences is that the necessary arrangements are made to meet the needs of the children at the lowest level in the class and the children at the highest level. For example, instead of presenting the same problem situation to children with high math performance by teachers repeatedly, it is necessary to create more challenging problem situations for them. On the other hand, care should be taken to present problems that do not reduce the self-efficacy of low-performing children and not to discriminate between high and low performance in the classroom (Clements & Sarama, 2020).

Family participation in early childhood settings is also important in terms of reducing individual differences. In the process of observing and evaluating individual differences, the teacher and the family need to move forward together, as well as carry out subsequent processes together (NAEYC, 2020). In their study, Bailey and Bulotsky-Shearer (2022) state that the cooperation of teachers and parents, especially in relation to the needs and strengths of children with individual differences, will positively impact children's learning behaviors related to mathematics. Similarly, Cook & Coley (2017) concluded that when teachers involve families in the educational process, their mathematics skills are positively affected. The cooperation with the parent will increase the awareness of parents about the mathematical development of children and will create an opportunity to support the child at home.

Peer learning also makes an important contribution to reducing individual differences between children. As the Montessori stated Montessori (1967): *"Our schools show that children of different ages help one another. The younger ones see what the older ones are doing and ask for explanations. They are quite ready to make this statement. These are readily given, and the instruction is really valuable, For the mind of a five year old is so much nearer than ours to the mind of a child of three The older ones are happy to be able to teach what they know. There are no inferiority complexes, but everyone achieves a healthy normality through the mutual exchange."* (Lillard, 2005, s. 192). For this reason, peer education is a frequently preferred way to learn mathematics (Harris, 2019). However, Wood & Frid (2005) noted that peer support is not alone and should be strengthened through interactive ways such as creating a discussion environment. Similarly, Tudge (1992), in his study examining the effects of peer support on reasoning in mathematics within the scope of convergent developmental domain (Vygotsky & Cole, 1978), states that it is important to develop a common understanding and conversations between children during peer support. In this direction, it is necessary to determine the cause of individual differences, to consider one-on-one, family, peer paths together according to the characteristics of the child. Supporting children developmentally with equal opportunities in education will contribute to reducing individual differences in terms of ensuring sustainability in educational achievement.

When teachers' achievements from mathematics education-oriented projects were analyzed, positive reactions of families, sharing information with colleagues, sharing information with academicians, and development of research skills were found. In high-quality mathematics programs, all stakeholders related to the children, such as the classroom, school, family, and society, should work together to create the necessary educational environment and resources for children during the educational process (NCTM, 2013). Family participation is systematically included in math-oriented projects. Therefore, there is an increase in the attitudes and perceptions of families; they are more open to cooperation (Silver et al., 2023; Ramani & Siegler, 2008). Projects are carried out in cooperation with colleagues and field experts; planning, preparation, and implementation sessions are held for common goals. These experiences help teachers gain more confidence about early mathematics and lead them to collaborative practices. Thus, teachers are encouraged to provide children with different experiences in early mathematics and develop their research skills (Fishman, et al., 2003).

Among the suggestions offered by pre-school teachers for quality early mathematics education were in-service training, teachers' openness to learning, family involvement, qualified undergraduate education, mathematics activity collection, e-Twinning projects, using technology effectively, following children individually and providing appropriate support, using natural materials, and using different techniques. When the related literature was examined, similar findings were obtained indicating that teachers need support other than undergraduate education related to mathematics education (Koç, 2017), teachers should receive qualified trainings that support their knowledge and practices (Orçan Kaçan & Halmatov, 2017), and they should receive in-service training in planning and evaluating activities (Pekince & Avcı, 2016). Teachers are the most important component in providing qualified early mathematics experiences (Stoll, 2015). Although rich environments for early mathematics prepared by teachers for children are an important indicator of qualified education, they are not enough on their own. The scope of mathematics is quite wide, and teachers should actively participate in the educational process in order to support and improve children's mathematical skills (Lee & Ginsburg, 2009). It can be said that the participation of teachers in high-quality, organized and applied early mathematics content trainings related to pedagogical field knowledge, individual differences, and mathematical thinking processes will enable them to organize the classroom environment with better quality. With the mathematics education to be given to teachers systematically, it can be ensured that they are aware of current learning methods and enrich the context of mathematics education.

This study is limited to the statements of teachers with project experience. Since it is carried out within the framework of the qualitative method, it may not be possible to generalize, so research can be carried out in which quantitative and qualitative methods are used together. Teachers with experience in mathematics-oriented projects were recruited to the study group, their views on mathematics education were examined, and they tried to be a guide for educators and researchers. Future studies can be conducted with pre-school teachers with low and high mathematics motivation or perception, and the reasons that prevent or increase the enrichment of mathematics in the educational environment can be revealed comparatively.

Availability of Data and Materials: The data that support the findings of this study are available on request from the corresponding author.

Ethics Committee Approval: Ethics committee approval was obtained from Aksaray University Human Research Ethics Committee (Approval no: 2023/03-71, Date: 25.04.2023).

Informed Consent: In the study, a written "Informed Consent Form" was obtained from the teachers before the interviews.

Peer-review: Externally peer-reviewed.

Author Contributions: Concept – F.A.B., İ.U.; Design – F.A.B., İ.U.; Supervision – İ.U.; Resources – F.A.B., İ.U.; Materials – F.A.B., İ.U.; Data Collection and/or Processing – F.A.B., İ.U.; Analysis and/or Interpretation – F.A.B., İ.U.; Literature Search – F.A.B., İ.U.; Writing Manuscript – F.A.B., İ.U.; Critical Review – F.A.B., İ.U.; Other – F.A.B., İ.U.

Declaration of Interests: The authors have no conflict of interest to declare.

Funding: The authors declared that this study has received no financial support.

References

Altan, R. Y., Genç, H., & Dağlioğlu, H. E. (2021). Türkiye'de okul öncesi dönemde matematik alanında yapılan çalışmalara ilişkin bir içerik analizi. *Opus International Journal of Society Researches*, *17*(33), 619–653. [CrossRef]

Akıncı-Coşgun, A., & Yılmaz, M. M. (2021). Planlamadan uygulamaya: Okul öncesi öğretmenlerinin matematik etkinliklerine yaklaşımı. *Journal of International Education Science*, *8*(29), 248–273.

Anthony, G., McLachlan, C., & Lim Fock Poh, R. (2015). Narrative assessment: Making mathematics learning visible in early childhood settings. *Mathematics Education Research Journal*, *27*(3), 385–400. [CrossRef]

Bailey, J., & Bulotsky-Shearer, R. J. (2022). Learning behaviors mediate associations between parent-reported peer play skills and literacy and mathematics skills for low-income preschool children. *Early Education and Development*, *33*(3), 523–540. [CrossRef]

Baki, A., & Karadeniz Hacısalihoğlu, M. (2013). Okul öncesi eğitim programının matematik uygulama sürecinden yansımalar. *Kastamonu Eğitim Dergisi*, *21*(2), 619–636.

Björklund, C., & Barendregt, W. (2016). Teachers' pedagogical mathematical awareness in Swedish early childhood education. *Scandinavian Journal of Educational Research*, *60*(3), 359–377. [CrossRef]

Björklund, C., Magnusson, M., & Palmér, H. (2020). Teachers' involvement in children's mathematizing – Beyond dichotomization between play and teaching. *European Early Childhood Education Research Journal*, *26*(4), 469–480. [CrossRef]

Björklund, C., van den Heuvel-Panhuizen, M., & Kullberg, A. (2020). Research on early childhood mathematics teaching and learning. *ZDM*, *52*(4), 607–619. [CrossRef]

Bodovski, K., & Farkas, G. (2007). Mathematics growth in early elementary school: The roles of beginning knowledge, student engagement, and instruction. *Elementary School Journal*, *108*(2), 115–130. [CrossRef]

Breive, S. (2019). Kindergarten teachers' orchestration of mathematical learning activities: The balance between freedom and structure. In Eleventh Congress of the European Society for Research in Mathematics Education (CERME11), (No. 06). Freudenthal Group; Freudenthal Institute; ERME.

Büyüköztürk, Ş., Kılıç-Çakmak, E., Akgün, Ö., Karadeniz, Ş., & Demirel, F. (2017). *Bilimsel araştırma yöntemleri*. Pegem Akademi.

Çakır, R., & Ergül, C. (2022). Düşük erken matematik başarısı gösteren çocukların çalışma belleği performanslarının incelenmesi. *Eğitim ve Bilim*, *47*(209). [CrossRef]

Çerezci, B. (2019). Early mathematics teaching profiles. *International Journal of Educational Research Review*, *4*(3), 288–302. [CrossRef]

Ceylan, M., & Aslan, D. (2023). Erken çocukluk dönemi matematik eğitiminde öğrenme rotaları yaklaşımı ve meb okul öncesi eğitim programının karşılaştırılması. *International Journal of Educational Studies in Mathematics*, *10*(1), 26–40. [CrossRef]

Chen, J. Q., & McCray, J. (2012). A conceptual framework for teacher professional development: The whole teacher approach. *NHSA Dialog*, *15*(1), 8–23. [CrossRef]

Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach*. Routledge.

Cook, K. D., & Coley, R. L. (2017). School transition practices and children's social and academic adjustment in kindergarten. *Journal of Educational Psychology*, *109*(2), 166–177. [CrossRef]

Creswell, J. W. (2023). *Nitel araştırma yöntemleri. Qualitative research methods* (M. Bütün, S. B. Demir Çeviri, Eds.). Siyasal Kitabevi.

Donlan, C. (2020). Book review of "an introduction to mathematical cognition" by C. Gilmore, SM Göbel, and M. Inglis. *Journal of Numerical Cognition*, *6*(2), 260–262. [CrossRef]

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. [CrossRef]

Ersan, O., & Rodriguez, M. C. (2020). Socioeconomic status and beyond: A multilevel analysis of TIMSS mathematics achievement given student and school context in Turkey. *Large-Scale Assessments in Education*, *8*, 1–32. [CrossRef]

Fırat, Z. S., & Dinçer, Ç. (2018). Comparing the opinions and practices of preschool teachers' about the use of natural mathematics language. *Turkish Online Journal of Qualitative Inquiry*, *9*(4), 312–341. [CrossRef]

Fishman, B. J., Marx, R. W., Best, S., & Tal, R. T. (2003). Linking teacher and student learning to improve professional development in systemic reform. *Teaching and Teacher Education*, *19*(6), 643–658. [CrossRef]

Gasteiger, H., & Benz, C. (2018). Enhancing and analyzing kindergarten teachers' professional knowledge for early mathematics education. *Journal of Mathematical Behavior*, *51*, 109–117. [CrossRef]

Gilmore, C., Göbel, S. M., & Inglis, M. (2018). *An introduction to mathematical cognition*. Routledge.

Hardy, J. K., & Hemmeter, M. L. (2019). Systematic instruction of early math skills for preschoolers at risk for math delays. *Topics in Early Childhood Special Education*, *38*(4), 234–247. [CrossRef]

Harris, K. I. (2019). Using peer-shared intervention strategies for promoting math explorations and critical thinking in early childhood inclusive classrooms. *International Journal of Contemporary Education*, *2*(1), 33–42. [CrossRef]

Karakuş, H., Akman, B., & Durmuşoğlu, M. (2022). Okul öncesi öğretmenlerinin matematik eğitimine ve sınıf içi uygulamalarına ilişkin görüşleri. *e-Kafkas Eğitim Araştırmaları Dergisi*, *9*(1), 171–193. [CrossRef]

Koç, D. (2017). *Okul öncesi dönemde matematik eğitimi: Öğretmen uygulamaları ve görüşleri üzerine bir durum çalışması* (Yayınlanmamış Yüksek Lisans Tezi). Uludağ Üniversitesi.

LeCompte, M. D., & Goetz, J. P. (1982). Problems of reliability and validity in ethnographic research. *Review of Educational Research*, *52*(1), 31–60. [CrossRef]

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, *34*(4), 37–45. [CrossRef]

Lillard, A. S. (2005). *Montessori the science behind the genius*. Oxford University Press.

Lombardi, C. M., & Dearing, E. (2021). Maternal support of children's math learning in associations between family income and math school readiness. *Child Development*, *92*(1), e39–e55. [CrossRef]

Merriam, S. B. (2015). *Nitel araştırma: Desen ve uygulama için bir rehber* (S. Turan, Çev.). Nobel Akademik Yayıncılık.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Milli Eğitim Bakanlığı Yenilik ve Eğitim Teknolojileri Genel Müdürlüğü. (2022). Retrieved September 3, 2023, from https://yegitek.meb.gov.tr/www/matematik-seferberligi-devam-ediyor/icerik/3400

Montessori, M. (1967). *The absorbent mind* (C. Claremont, Trans.). Delta.

NAECS/SDE (2003). *Joint Position Statement. Early Childhood Curriculum, Assessment, and Programme Evaluation: Building an Effective, Accountable System in Programs for Children Birth Through Cape 8*. http://www.naeyc.org.

National Association for the Education of Young Children (2020). *Professional standards and competencies for early childhood educators*.

National Association for the Education of Young Children, & National Council of Teachers of Mathematics (NAEYC & NCTM) (2010). *Position statement. Early childhood mathematics: Promoting good beginnings*. [CrossRef]

National Council of Teachers of Mathematics (2013). *Mathematics in early childhood learning*.

Orçan Kaçan, M., & Halmatov, M. (2017). Türkiye'de uygulanan okul öncesi eğitim programında matematik: Planlama ve uygulama. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi*, *42*(42), 149–161. [CrossRef]

Pekince, P., & Avcı, N. (2016). Okul öncesi öğretmenlerinin erken çocukluk matematiği ile ilgili uygulamaları: Etkinlik planlarına nitel bir bakış. *Kastamonu Eğitim Dergisi*, *24*(5), 2391–2408.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development*, *79*(2), 375–394. [CrossRef]

Reikerås, E., & Salomonsen, T. (2019). Weak mathematical skills at an early age: Persistent or temporary? Children with weak mathematical skills and their development from toddlers to preschoolers. *Early Child Development and Care*, *189*(4), 670–682. [CrossRef]

Silver, A. M., Chen, Y., Smith, D. K., Tamis-LeMonda, C. S., Cabrera, N., & Libertus, M. E. (2023). Mothers' and fathers' engagement in math activities with their toddler sons and daughters: The moderating role of parental math beliefs. *Frontiers in Psychology*, *14*, 1124056. [CrossRef]

Sonnenschein, S., Grossman, E. R., & Grossman, J. A. (2021). US parents' reports of assisting their children with distance learning during Covid-19. *Education Sciences*, *11*(9), 501. [CrossRef]

Sprenger, P., & Benz, C. (2020). Children's perception of structures when determining cardinality of sets—Results of an eye-tracking study with 5-year-old children. *ZDM*, *52*(4), 753–765. [CrossRef]

Stoll, J. A. (2015). *An exploratory study of preschool teachers' perceived knowledge, behaviors and attitudes/beliefs regarding the National Council of Teachers of Mathematics (NCTM) process standards*. Kent State University.

TantekİN Erden, F., & Tonga, F. E. (2020). Okul öncesi öğretmenlerinin matematik eğitimine ilişkin görüşleri: Matematik öğretimi, cinsiyet farklılıkları, öğretmenin rolü. *Balıkesir Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, *23*(44), 845–862. [CrossRef]

Tudge, J. R. H. (1992). Processes and consequences of peer collaboration: A Vygotskian analysis. *Child Development*, *63*(6), 1364–1379. [CrossRef]

Tzekaki, M. (2020). Mathematical activity in early childhood and the role of generalization. In Mathematics Education in the Early Years: Results from the POEM4 Conference, 2018 (pp. 301–313). Springer International Publishing.

Tzekaki, M., & Papadopoulou, E. (2017). *Teaching intervention for developing generalization in early childhood: The case of measurement*. In CERME 10.

van Oers, B., & Poland, M. (2012). Promoting abstract thinking in young children's play. In B. van Oers (Ed.), *Developmental Education for young children. Concept, practice, and implementation* (pp. 121–136). Springer. [CrossRef]

Vogler, A. M. (2019). Chances and obstacles of 'indirect'learning processes in situations with preschool teachers. In Eleventh congress of the European Society for Research in Mathematics Education (CERME11) (No. 29). Freudenthal Group; Freudenthal Institute; ERME.

Vygotsky, L. S., & Cole, M. (1978). *Mind in society: Development of higher psychological processes*. Harvard university press.

Wood, K., & Frid, S. (2005). Early childhood numeracy in a multiage setting. *Mathematics Education Research Journal*, *16*(3), 80–99. [CrossRef]

Yazıcıoğlu, T., & Akdal, D. (2023). An analysis of the effects of the Early Mathematics Intervention Program on early mathematics skills of pre-school children at risk. *Participatory Educational Research*, *10*(1), 237–250. [CrossRef]

Yazlık, D. Ö., & Öngören, S. (2018). Okul öncesi öğretmenlerinin matematik etkinliklerine ilişkin görüşlerinin ve sınıf içi uygulamalarının incelenmesi. *AHI Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi*, *19*(2), 1264–1283. [CrossRef]

Yıldırım, A., & Şimşek, H. (2016). *Sosyal bilimlerde nitel araştırma yöntemleri* (11.baskı). Seçkin yayıncılık.